Coordinating Supply Chains with a Credit Mechanism

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Abstract: This paper studies the supply chain coordination with a trade credit under symmetric and asymmetric information, where the retailer has an individual profit target from the business and the vendor is the decision-maker of the supply chain. We propose a coordination mechanism through credit contracts and show that a win-win outcome is achieved by redistributing the cost savings from coordination mechanism under certain constraints. Numerical examples are given to illustrate our results.

Keywords: *supply chain coordination; trade credit; contract; information asymmetry.*

Mathematics Subject Classification (2010): 90B05.

1 Introduction

In the current competitive business environment, trade credit has been widely used and represents an important proportion of firms finance. Rajan and Zingales reported that accounts payable amounted to 15% of the assets for a sample of nonfinancial U.S. firms on Global Vantage while debt in current liabilities accounted for just 7.4% [1]. In China, it was once overused, e.g., powerful buyers like WalMart, Carrefour and Gome used to allow delayed payments to their vendors for as long as one year, so that the government has made a law to ban buyers from delaying payment for more than two months. Since Goyal first developed an economic order quantity (EOQ) model under the conditions of permissible delay in payments [2], there is a great deal of literature dealing with a variety

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of situations such as shortages allowed, partial backlogging, credit-linked demand/order quantity, deterioration etc. Chung utilized the discounted cash-flows (DCF) approach to analyze the optimal inventory policy in the presence of the trade credit [3]. Jamal et al. extended Goyal's model to consider the deteriorating items and to allow for shortages [4]. Teng amended Goyal's model by considering the difference between unit price and unit cost, then established an easy analytical closed-form to the problem [5]. Song and Cai studied the payment time of the retailer and the length of the inventory cycle under permitted delay of payment by wholesaler and derived the optimal joint solution [6]. Chung and Huang extended Goyal's model to the case that the units are replenished at a finite rate [7]. Hu and Liu investigated retailer's optimal replenishment policy under conditions of permissible delay in payments and allowable shortages within the economic production quantity (EPQ) framework [8]. Ouyang et al. developed an inventory model with non-instantaneous receipt where the supplier provides not only a permissible delay but also a cash discount for the retailer [9]. In [10], Ouyang et al. investigated a generalized inventory model for deteriorating items under the delay in payments linked to order quantity. Liang and Zhou developed a two-warehouse inventory model for deteriorating items under permissible delay in payment [11]. With purchasing cost depending on the delay in payments and the order quantity, Krichen et al. proposed a solution approach that generates stable coalition structures for the retailers [12]. Jaggi et al. investigated the impact of credit-linked demand on the retailers optimal replenishment policy under two levels of trade credit policy [13]. Annadurai and Uthayakumar further extended the work of Jaggi et al. [13], by including deteriorating items and backlogging [14].

To the best of our knowledge, Jaber and Osman first used trade credit as a mechanism to coordinate a two-level supply chain [15]. In [16], Sarmah et al. investigated a coordination problem in a single-manufacturer and multiple heterogeneous buyers situation with a credit option. Recently, Duan et al. studied the supply chain coordination policy by delay in payments for the products with fixed lifetime [17]. However, information asymmetry is not considered by the above-cited literature. Luo and Zhang studied the benefit of coordinating a supply chain by a credit contract under both symmetric and asymmetric information [18]. But in their analysis, the vendor gets all the cost savings from the coordination regardless of the buyer's benefits, and the supply chain cannot be coordinated under asymmetric information. However, each member of a supply chain wants a certain fixed amount of cost reduction from the business and hence parties will not be interested in coordinating if their target profit is not achieved. In this paper, we proposed a trade credit contract to coordinate the supply chain from the business where both the parties have their cost targets. Considering credit policy as coordination mechanism between the two parties, our objective is to derive the optimal credit periods under symmetric and asymmetric information, respectively. In addition, we show that using trade credit can coordinate the supply chain in cases with both the symmetric and asymmetric information under some constraints.

2   Model Formulation Under Symmetric Information

2.1   Notation and assumptions

The following notations and assumptions are adopted throughout this paper.

Notation:

S_r	retailer's ordering cost per order;
S_v	vendor's setup cost per production;
D	market demand rate;
h	retailer's inventory holding cost per unit time;
I_r, I_v	retailer's and vendor's unit capital opportunity;
K	integer lot size multiplier per cycle;
$M(K)$	the credit period offered by the vendor;
Q	order quantity of the buyer.

Assumption 2.1 Demand rate is known and constant.

Assumption 2.2 The vendor follows a lot-for-lot manufacturing policy.

Assumption 2.3 Both production and replenishment are instantaneous.

Assumption 2.4 Shortages are not allowed.

2.2   The trade credit under full information

In the absence of any coordination, it is easy to obtain that the retailer's optimal lot size is simply the EOQ, i.e., $Q_r = \sqrt{\frac{2DS_r}{h+I_r}}$, and the retailer's corresponding cost per unit time is

$$TC_r = \sqrt{2DS_r(h + I_r)}. \tag{1}$$

Under the lot-for-lot system, the vendor's cost including only the setup cost is $TC_v = \frac{DS_v}{Q_r}$. For the whole supply chain, the joint cost with coordination is

$$TC(Q) = \frac{D(S_r + S_v)}{Q} + \frac{(h + I_r)Q}{2}. \tag{2}$$

Minimizing (2) yields the optimal lot size for the whole system

$$Q_c = \sqrt{\frac{2D(S_r + S_v)}{h + I_r}}, \tag{3}$$

which is obvious larger than Q_r. Hence, in order to entice the buyer to alter his current EOQ by a factor $K(K > 0)$, i.e., the retailer's new ordering quantity is KQ_r, the vendor offers a delay period $M(K)$ (the delay period M is dependent on the retailer's order size) to the buyer to compensate the retailer for his increased inventory cost, and possibly provides an additional saving, such that the vendor can benefit from higher order quantity. It is assumed that the buyer is willing to coordinate as long as his target cost is no larger than $[1 - G(K)]TC_r$, where $G(K)(\geqslant 0)$ is the reduction factor of retailer's target cost from supply chain coordination , such that the retailer can also benefit from the supply chain coordination. For simplification, we assume that $G(K)$ is a linear function of K, i.e, $G(K) = \alpha K + \beta$.

Under the above trade credit contract, the buyer's inventory cost per unit time, denoted by $\widetilde{TC}_r(Q, K)$, is

$$\widetilde{TC}_r(Q, K) = \frac{DS_r}{KQ_r} + \frac{(h + I_r)KQ_r}{2} - DI_rM(K), \tag{4}$$

and the vendor's cost per unit time is

$$\widetilde{TC}_v(Q,K) = \frac{DS_v}{KQ_r} + DI_vM(K). \tag{5}$$

For achieving the retailer's target cost, it is obvious that the trade credit should satisfy

$$\frac{DS_r}{KQ_r} + \frac{(h+I_r)KQ_r}{2} - DI_rM(K) \leqslant (1-\beta-\alpha K)TC_r. \tag{6}$$

Simplifying the above inequality, we have

$$M(K) \geqslant \frac{TC_r}{2DI_r}\left[\frac{1}{K} + K - 2(1-\beta-\alpha K)\right]. \tag{7}$$

Next, we will determine the vendor's optimal lot size. From (5), we can see that the vendor's cost is minimized only if the trade credit gets the smallest value. Hence, substituting $M(K) = \frac{TC_r}{2DI_r}\left[\frac{1}{K} + K - 2(1-\beta-\alpha K)\right]$ into (5), yields

$$\widetilde{TC}_v(Q,K) = \frac{DS_v}{KQ_r} + \frac{I_vTC_r}{2I_r}\left[\frac{1}{K} + K - 2(1-\beta-\alpha K)\right]. \tag{8}$$

It is easy to verify that $\widetilde{TC}_v(Q,K)$ in (8) is convex in K, so the optimal value of K is determined by the first-order condition as follows

$$\frac{\partial \widetilde{TC}_v(Q,K)}{\partial K} = \frac{(1+2\alpha)I_vTC_r}{2I_r} - \frac{TC_r}{2K^2}\left(\frac{S_v}{S_r} + \frac{I_v}{I_r}\right) = 0. \tag{9}$$

From (9), we get

$$K^* = \sqrt{\frac{\frac{S_v}{S_r}\frac{I_r}{I_v}+1}{1+2\alpha}}.$$

Then $Q_c = K^*Q_r$ if $\alpha^* = \frac{S_v(I_r-I_v)}{2(S_r+S_v)I_v}$. Furthermore, from the perspective of vendor, it is obvious that β should satisfy

$$\frac{DS_v}{KQ_r} + \frac{I_vTC_r}{2I_r}\left[\frac{1}{K^*} + K^* - 2(1-\beta-\alpha^*K^*)\right] \geqslant \frac{DS_v}{Q_r},$$

if supply chain coordination can be achieved, i.e., $\beta \geqslant 1-\left[(1+2\alpha^*)K^* - \frac{S_vI_r}{2S_rI_v}\right]$. Hence, we have the following proposition.

Proposition 1 *If*

$$\alpha^* = \frac{S_v(I_r-I_v)}{2(S_r+S_v)I_v}, \tag{10}$$

then the proposed trade credit contract can not only achieves the supply chain coordination but achieve the win-win outcome for any given

$$\beta \geqslant 1-\left[(1+2\alpha^*)K^* - \frac{S_vI_r}{2S_rI_v}\right]. \tag{11}$$

In reality, according to the firm's bargaining powers, the benefits allocation can be achieved by changing β, e.g., a simple effective way to set the benefits allocated is to divide them equitably between the two firms.

3 Model Formulation Under Asymmetric Information

In this section, we consider the situation that the capital cost of the buyer is his private information. Under asymmetric information, the vendor will offer a menu of contracts for the sake of stimulating the buyer to reveal his information via contract selection. This so-called screening game in the game theory, where the vendor is the first to move, can derive the optimal contract by using the approach developed in [19].

We assume that the vendor knows the random variable I_r characterized by a prior distribution $F(I_r)$ and the corresponding density function $f(I_r)$ on its domain $[\underline{I_r}, \overline{I_r}]$, where $0 \leqslant \underline{I_r} < \overline{I_r} < \infty$. In the case of information asymmetry, the vendor now offers a menu of contracts $\{K, M(K)\}$. Then the buyer chooses a specific pair from the contract menu, and the vendor can infer buyer's I_r from his selection.

Based on the above arguments, we can present the vendor's contract design problem as the following optimization program

$$\min_{M, K} \widetilde{TC}_v^A = \int_{\underline{I_r}}^{\overline{I_r}} \left[\frac{DS_v}{KQ_r} + DI_v M(K) \right] f(I_r) dI_r \tag{12}$$

such that

$$\frac{DS_r}{KQ_r} + \frac{(h + I_r)KQ_r}{2} - DI_r M(K)] \leqslant \frac{DS_r}{\tilde{K}Q_r} + \frac{(h + I_r)\tilde{K}Q_r}{2} - DI_r M(\tilde{K}), \tag{12a}$$

$$\frac{DS_r}{KQ} + \frac{(h + I_r)KQ}{2} - DwM(K) \leqslant (1 - \beta - \alpha K)TC_r^0, \tag{12b}$$

$$\underline{I_r} \leqslant I_r \leqslant \overline{I_r}. \tag{12c}$$

The constraint condition in (12a) indicates that the retailer will choose the optimal contract menu to minimize his cost, and the constraint condition in (12b) requires that the buyer's cost with trade credit must be not higher than his reservation cost, for ensuring his participation. TC_r^0 is an exogenous variable that is retailer's reservation cost (e.g., it can be set as the retailer's cost without coordination).

The following Proposition 2 gives the vendor's optimal menu of credit contracts.

Proposition 2 *When the retailer has the personal information about his capital cost I_r, the vendor's optimal contract menu is*

$$M_A^* = \frac{1}{DI_r} \left[\frac{(h + I_r)K_A^* Q_r}{2} + \frac{DS_r}{K_A^* Q_r} \right] - \frac{(1 - \beta - \alpha K_A^*)TC_r^0}{D\overline{I_r}}, \tag{13}$$

where K_A^* is given as follows

$$K_A^* = \begin{cases} \frac{1}{Q_r}\sqrt{\frac{2DS_v I_r + 2DS_r[I_v - F(I_r)/f(I_r)]}{h[I_v - F(I_r)/f(I_r)] + I_r I_v + 2\alpha I_r TC_r^0/Q_r}}, \text{if} I_v \geqslant \\ \max\left\{ \frac{hS_v}{S_r} - \frac{2\alpha TC_r^0}{Q_r}, \frac{hF(I_r)/f(I_r) - 2\alpha I_r TC_r^0/Q_r}{h + I_r} \right\} \\ \text{or } I_v \leqslant \min\left\{ \frac{F(I_r)}{f(I_r)} - \frac{S_v I_r}{S_r}, \frac{hS_v}{S_r} - \frac{2\alpha TC_r^0}{Q_r}, \frac{hF(I_r)/f(I_r) - 2\alpha I_r TC_r^0/Q_r}{h + I_r} \right\}; \\ \frac{1}{Q_r}\sqrt{\frac{2DS_r}{h}}, \text{otherwise.} \end{cases} \tag{14}$$

Proof. Let $U(I_r) = \frac{DS_r}{KQ_r} + \frac{(h+I_r)KQ_r}{2} - DI_rM(K)$, then $U'(I_r) = -\frac{DS_rK'}{K^2Q_r} + \frac{(h+I_r)Q_rK'}{2} + \frac{KQ_r}{2} - DI_rM' - DM$. Hence, we have

$$U(I_r) = U(\overline{I_r}) - \int_{I_r}^{\overline{I_r}} \left(\frac{KQ_r}{2} - DM \right) dI_r. \tag{15}$$

Furthermore, the constraint in (12a) is equal to

$$\left. \frac{dU(\tilde{I}_r)}{d\tilde{I}_r} \right|_{\tilde{I}_r = I_r} = 0, \tag{16}$$

$$\left. \frac{d^2U(\tilde{I}_r)}{d\tilde{I}_r^{\,2}} \right|_{\tilde{I}_r = I_r} \geqslant 0. \tag{17}$$

That is, the buyer will incur the lowest cost when he chooses $\tilde{I}_r = I_r$. It follows from (16)–(17) that

$$\frac{DS_rK'}{K^2Q_r} - \frac{(h+I_r)Q_rK'}{2} + DI_rM' = 0, \tag{18}$$

$$-\frac{2DS_r(K')^2}{K^3Q_r} + \frac{DS_rK''}{K^2Q_r} - \frac{(h+I_r)Q_rK''}{2} + DI_rM'' \geqslant 0. \tag{19}$$

Differentiating both sides of (18) with respect to I_r yields

$$-\frac{2DS_r(K')^2}{K^3Q_r} + \frac{DS_rK''}{K^2Q_r} - \frac{(h+I_r)Q_rK''}{2} - \frac{Q_rK'}{2} + DI_rM'' + DM' = 0. \tag{20}$$

From (19) and (20), we obtain

$$DM' - \frac{Q_rK'}{2} \leqslant 0. \tag{21}$$

Since the objective function $\widetilde{TC}_v^A$ can be rewritten as

$$\widetilde{TC}_v^A = \int_{\underline{I_r}}^{\overline{I_r}} \left\{ \frac{D(A_v + A_r)}{KQ_r} + D(I_v - I_r)M(K) + \frac{(h+I_r)KQ_r}{2} - U(I_r) \right\} f(I_r)dI_r, \tag{22}$$

the vendor's expected cost is decreasing with $U(I_r)$. Hence the condition $U(\overline{I_r}) = (1 - \beta - \alpha K)TC_r^0$ must be satisfied for any given K.

Under the above discussion, the optimal problem in (12) can be simplified as

$$\min_{M,K} \widetilde{TC}_v^A = \int_{\underline{I_r}}^{\overline{I_r}} \left\{ \frac{D(S_v + S_r)}{KQ_r} + D(I_v - I_r)M + \left[\frac{(h+I_r)Q_r}{2} + \alpha TC_r^0 \right] K \right.$$

$$\left. + \left(\frac{KQ_r}{2} - DM \right) \frac{F(I_r)}{f(I_r)} \right\} f(I_r)dI_r - (1 - \beta)TC_r^0 \tag{23}$$

such that

$$DM' - \frac{Q_r K'}{2} \leqslant 0, \tag{23a}$$

$$\underline{I_r} \leqslant I_r \leqslant \overline{I_r}. \tag{23b}$$

Next, we first neglect the constraint conditions. Then the optimal contract must satisfy the following first-order condition with respect to K

$$-\frac{D(S_v + S_r)}{K^2 Q_r} + D(I_v - I_r)\frac{dM}{dK} + \frac{(h + I_r)Q_r + 2\alpha TC_r^0}{2} + \left(\frac{Q_r}{2} - D\frac{dM}{dK}\right)\frac{F(I_r)}{f(I_r)} = 0. \tag{24}$$

Since (18) implies that $\frac{dM}{dK} = \frac{1}{DI_r}\left[\frac{(h+I_r)Q_r}{2} - \frac{DS_r}{K^2 Q_r}\right]$, we have

$$M = \frac{1}{DI_r}\left[\frac{(h + I_r)KQ_r}{2} + \frac{DS_r}{KQ_r}\right] - \frac{(1 - \beta - \alpha K)TC_r^0}{D\overline{I_r}}. \tag{25}$$

Combining (24) and (25) leads to the optimal order multiple

$$K_A^* = \frac{1}{Q_r}\sqrt{\frac{2DS_v I_r + 2DS_r[I_v - F(I_r)/f(I_r)]}{h[I_v - F(I_r)/f(I_r)] + I_r I_v + 2\alpha I_r TC_r^0/Q_r}}. \tag{26}$$

Now, taking into account the neglected constraint in (23a) leads to

$$K^2 \leqslant \frac{2DS_r}{hQ_r^2}. \tag{27}$$

Solving inequity (27), we get

$$I_v \geqslant \max\left\{\frac{hS_v}{S_r} - \frac{2\alpha TC_r^0}{Q_r}, \frac{hF(I_r)/f(I_r) - 2\alpha I_r TC_r^0/Q_r}{h + I_r}\right\} \tag{28}$$

or

$$I_v \leqslant \min\left\{\frac{F(I_r)}{f(I_r)} - \frac{S_v I_r}{S_r}, \frac{hS_v}{S_r} - \frac{2\alpha TC_r^0}{Q_r}, \frac{hF(I_r)/f(I_r) - 2\alpha I_r TC_r^0/Q_r}{h + I_r}\right\}. \tag{29}$$

When inequalities (28)–(29) are not fulfilled, K_A^* does not satisfy the constraint in (23a) and hence it is not the optimal solution of optimization problem (12). But we can easily prove that $\widetilde{TC}_v^A$ is decreasing on the interval $\left[0, \frac{1}{Q_r}\sqrt{\frac{2DS_r}{h}}\right]$ (see [18]), and hence the vendor has the minimum cost at $K_A^* = \frac{1}{Q_r}\sqrt{\frac{2DS_r}{h}}$ in this case. $\square$

From Proposition 2, if

$$I_v \leqslant \min\left\{\frac{F(I_r)}{f(I_r)} - \frac{S_v I_r}{S_r}, \frac{hS_v}{S_r} - \frac{2\alpha_A^* TC_r^0}{Q_r}, \frac{hF(I_r)/f(I_r) - 2\alpha_A^* I_r TC_r^0/Q_r}{h + I_r}\right\}$$

or

$$I_v \geqslant \max\left\{\frac{hS_v}{S_r} - \frac{2\alpha_A^* TC_r^0}{Q_r}, \frac{hF(I_r)/f(I_r) - 2\alpha_A^* I_r TC_r^0/Q_r}{h + I_r}\right\}$$

for all $I_r \in [\underline{I_r}, \overline{I_r}]$, then the supply chain can be coordinated by choosing an appropriate value of β, where

$$\alpha_A^* = \frac{Q_r}{I_r TC_r^0} \left\{ \frac{DS_v I_r + DS_r[I_v - F(I_r)/f(I_r)]}{1 + S_v/S_r} - \frac{h}{2}\left[I_v - \frac{F(I_r)}{f(I_r)}\right] - \frac{I_r I_v}{2} \right\}. \quad (30)$$

As a result, the joint menu of contracts $(\alpha_A^*, K_A^*, M(K_A^*))$ achieves chain coordination between the retailer and the vendor.

4   Numerical Examples

In this section, we first study a full information case to examine the division of surplus generated due to coordination. The basic parameters are set as follows: $D = 1500$ units/year, $S_v = 2000$ /order, $S_r = 500$ /order, $h = 10$ /unit/year.

Example 1. If $I_r = 4$ and $I_v = 3$, then by Proposition 1, $(\alpha^*, K^*) = (0.13, 2.24)$. For given $\beta = 0.02$, the retailer's cost and the vendor's cost are 3124.66 and 6366.41, i.e., decrease is 46.66% and 43.96% from without coordination, respectively.

Example 2. If $I_r = 5$ and $I_v = 8$, then by Proposition 1, $(\alpha^*, K^*) = (-0.15, 2.24)$. For given $\beta = 0.45$, the retailer's cost and the vendor's cost are 4199.87 and 7705.19, i.e., decrease is 12.94% and 23.12% from without coordination, respectively.

Example 3. Assume that $I_v = 2$ and $I_r \sim U(2,6)$ (uniformly distribution), then by Proposition 2, the optimal value of K is $K_A^* = \sqrt{1 + 0.1I_r}$ and the corresponding credit menu is $M_A^* = \sqrt{5/3}(0.1 + 2/I_r) + 0.02\sqrt{1 + 0.1I_r} - 0.34$ for given I_r. Hence the vendor's cost is 6971.33. The costs for the vendor and the retailer decrease are 15.63% and 20.91% from without coordination, respectively.

Example 4. Assume that $I_v = 10$ and $I_r \sim U(3,5)$, then by Proposition 2, the joint menu of contracts $(\alpha_A^*, K_A^*, M(K_A^*))$, which is determined by (13), (14) and (30), can coordination the supply chain. For fixed $\beta = 0.12$ and $TC_r^0 = 4000$, the costs for the vendor and the retailer decrease are 25.43% and 16.17% from without coordination, respectively.

5   Conclusion

In this paper, through credit contracts, we study the division of benefits sharing for the supply chain coordination from the business under both symmetric and asymmetric information. To some extent, the results of this paper may be applied to some practice business. The wide usage of trade credit has shown that it can effectively reduce the cost of supply chain members. In real practice, the trade credit policy may also be more attractive than other policies like quantity discount to the retailer. In addition, by using a quantity-dependent trade credit to entice the retailer to alter his order quantity, the vendor can get much more benefits from the supply chain coordination, and the vendor also prefers such a policy when he is financially strong.

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