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PERSONAGE IN SCIENCE

Professor Taro Yoshizawa

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Taking into account the great significance of Yoshizawa's work for modern development of nonlinear dynamics and systems theory, the Editorial Board of the journal is publishing a sketch of his life, a brief survey of main directions of his scientific activity and a list of his work published in 1950–1997.

1 Biographical Sketch

Professor Taro Yoshizawa was born in Osaka on August 18, 1919. He received the degree of Bachelor of Science in Mathematics at Kyoto Imperial University where he graduated in December 1941. In August, 1949 he became assistant professor in mathematics at Kyoto University. In August, 1958 he was conferred a Doctor's degree in Mathematics by Kyoto University. He was assigned full professor in mathematics at Nihon University in April, 1959. He joined Tohoku University in August, 1965 and served there as professor of mathematics until his retirement in March, 1983. After that he was with Okayama University of Science as professor until March, 1996.

The main sphere of his scientific interests in mathematics was the stability of differential equations. Starting from September, 1959 he made a two year visit to the Research Institute for Advanced Studies established by Solomon Lefschetz in Baltimore, Md., USA. Since that time until as late as a few months before his death he played central roles in many international conferences and continued to be one of the world leaders in the stability theory of differential equations.

In April, 1993, in recognition of his merits he was decorated with the Third Order of the Rising Sun.

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2 Main Directions of Scientific Activity

Professor Yoshizawa was conducting his scientific research in the period of tremendous upgrowth of the theory of differential and functional differential equations due to its wide application in science and technology. Coming after the brilliant work of Liapunov (1892), the papers by K.P. Persidsky (1946), N.N. Krassovsky (1959), V.I. Zubov (1957), H.A. Antosiewicz (1958), L. Cesari (1963), S. Lefschetz (1963) and others exerted a strong influence on the formation of his scientific interests and determined the directions of his research. We now outline briefly the main avenues of his investigations.

2.1 General questions of the theory of differential equations

This direction was developed in papers [1, 2, 6, 7, 20]. Main results obtained before 1967 were summarized in monograph [33] (in Japanese). The series of works on limiting equations (see [35, 38, 39, 40] presented profound results in the analysis of nonautonomous systems in terms of limiting equations. Also, in these papers conditions of uniform asymptotic stability were established, eventual properties of solutions were studied, converse theorems on bounded properties of solutions of nonautonomous systems were proved as well as theorems on their stability and attraction. Some of these results were included into the generalizing monograph by Kato, A.A. Martynyuk, and A.A. Shestakov, Stability of Motion of Nonautonomous Systems (Method of Limiting Equations), Amsterdam: Gordon and Breach Publishers, 1996.

2.2 Liapunov stability and boundedness of solutions

Papers [9, 11–19, 28] deal with the investigations in this direction. Yoshisawa's monograph "Stability Theory by Liapinov's Second Method" (see [29] in the list of publications below) which followed the famous monographs by V.I. Zubov and N.N. Krassovsky proved to be the most often cited one in the English language literature on the important developments of the theory of stability of motion. Starting with the definitions of stability and boundedness of solutions for nonlinear systems, Yoshizawa set out basic theorems on stability and boundedness of solutions in terms of existence of Liapunov functions with appropriate properties. Completeness of these investigations is supported by the proofs of the converse theorems, i.e. the results showing existence of Liapunov functions with certain properties for certain types of stability of the zero solution.

2.3 Perturbed systems

Main results obtained by Yoshizawa in this direction were published in [24, 25] and then developed in Chapter 6 of monograph [29]. Namely, basing on some results by Gorshin (1936) and Malkin (1944), he proved a series of theorems on stability under persistent perturbations and studied behavior of solutions of perturbed systems both for systems of ordinary differential equations and for equations with small parameter. He also analyzed asymptotic properties of solutions near integral manifolds having developed thereby the results of Hale (1963) and Bogolyubov and Mitropolsky (1963).

2.4 Existence theorems for periodic solutions and almost periodic solutions

This direction is covered by papers [8, 25, 31, 32, 37]. Based on fixed point theorems, Yoshizawa established existence conditions for periodic solutions, which were the gen-

eralizations of some results of Massera (1950), and determined existence conditions for bounded solutions. These results were obtained by means of two Liapunov functions satisfying special conditions. In order to establish existence conditions for almost periodic solutions he applied radially unbounded Liapunov functions to the theory of asymptotic stability. Monograph [37] summarizes Yoshizawa's results obtained up to 1975.

2.5 Theory functional differential equations

Yoshizawa's papers [25, 27, 41-43, 44, 45] refer to this direction of investigations. To prove theorems on existence of solutions of functional differential equations he employed the method of Liapunov-Krassovsky functionals. General results on stability were also proved based on functionals of special type. As an extension of the results of LaSalle (1960), Yoshizawa studied asymptotic behavior of solutions of autonomous systems. He also established boundedness conditions for solutions, including equations with persistent perturbations. For periodic and almost periodic systems of equations the conditions of solution existence, stability and boundedness were obtained. For a survey and developments of some Yoshizawa's ideas and approaches see the monograph by A.Burton, Stability and Periodic Solutions of Ordinary and Functional Differential Equations, Orlando: Academic Press, Inc., 1985.

It should be noted that numerous reports made by Prof. Yoshizawa at international conferences and symposia always drew the audience and generated a keen interest.

Taro Yoshizawa was named professor emeritus from Tohoku University. He was one of the former editors of the Tohoku Mathematical Journal. He served as member of the board of directors of the Mathematical Society of Japan as well as other important committees.

Professor Taro Yoshizawa passed away in Kyoto on October 7, 1996.

3 The Life of a Teacher

Professor Yoshizawa was a leading mathematician, speaker, and writer in the broad area of stability theory of differential equations. But he will be remembered most of all for his teaching and mentoring former students and junior colleagues. While we have been unable to find a complete list of his doctoral students and the year in which they received their degrees we would mention the following former students and the year in which their first paper on differential equations was reviewed in the Mathematical Reviews:

1. Tetsuo Furumochi (1971)
2. Yoshihiro Hamaya (1989)
3. Yoshiyuki Hino (1970)
4. Takashi Kaminogo (1975)
5. Junji Kato (1962)
6. Satoru Murakami (1982)
7. Toshiki Naito (1970)
8. Fumio Nakajima (1971).

We have mentioned the major monographs by Prof. Yoshizawa including [29] on Liapunov theory and [37] on periodic and almost periodic solutions of differential equations. His teaching in these areas is strongly reflected in the subsequent work of all of his students. In fact, a logical continuation of his stability theory in [29] is seen in the major monograph by three of his students:

Y. Hino, S. Murakami, and T. Naito, Functional Differential Equations with Infinite Delay, Springer, Tokyo, 1991.

In the same way a logical continuation of his almost periodic theory in [37] is seen in the impressive monograph with two of his students as coauthors:

Y. Hino, T. Naito, Nguyen Van Minh, and Jong Son Shin, Almost Periodic Solutions of Differential Equations in Banach Spaces, Taylor and Francis, London and New York, 2002.

Professor Yoshizawa travelled extensively to meetings all over the world and was almost always accompanied by his wife and some of his former doctoral students. He maintained a life long relationship with his former students and was highly revered by all of them. His life as a teacher is a model which we could all try to emulate.

4 List of Selected Works of T. Yoshizawa

1. On the uniqueness of solutions of a system of ordinary differential equations. *Mem. Coll. Sci. Univ. Kyoto. Ser. A.* **26** (1950), 19–29. (with Kyuzo Hayashi)

2. New treatise of solutions of a system of ordinary differential equations and its application to the uniqueness theorems. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **26** (1951), 225–233. (with Kyuzo Hayashi)

3. Note on the non-increasing solutions of $y'' = f(x, y, y')$. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **27** (1952), 153–162.

4. On the evaluation of the derivatives of solutions of $y'' = f(x, y, y')$. *Mem. Coll. Sic. Univ. Kyoto. Ser. A. Math.* **28** (1953), 27–32.

5. Note on the boundedness of solutions of a system of differential equations. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **28** (1954), 293–298.

6. On the non-linear differential equation. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **28** (1954), 133–141.

7. On the convergence of solutions of the non-linear differential equation. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **28** (1954), 143–151.

8. Note on the existence theorem of a periodic solution of the non-linear differential equation. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **28** (1954), 153–159.

9. On the stability of solutions of a system of differential equations. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **29** (1955), 27–33.

10. Note on the solutions of a system of differential equations. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **29** (1955), 249–273.

11. Note on the boundedness and the ultimate boundedness of solutions of $x' = F(t, x)$. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **29** (1955), 275–291.

12. On the necessary and sufficient condition for the uniform boundedness of solutions of $x' = F(t, x)$. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **30** (1957), 217–226.

13. Appendix to the paper "Note on the boundedness and the ultimate boundedness." Mem. *Coll. Sci. Univ. Kyoto. Ser. A. Math.* **30** (1957), 91–103.

14. Note on the equi-ultimate boundedness of solutions of $x' = F(t, x)$. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **31** (1958), 211–217.

15. On the equiasymptotic stability in the large. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **32** (1959), 171–180.

16. Liapunov's function and boundedness of solutions. *Funkcial. Ekvac.* **2** (1959), 71–103.

17. Stability and boundedness of systems. *Arch. Rational Mech. Anal.* **6** (1960), 409–421.

18. Existence of a bounded solution and existence of a periodic solution of the differential equation of the second order. *Mem. Coll. Sci. Univ. Kyoto. Ser. A. Math.* **33** (1960/1961), 301–308.

19. Liapunov's function and boundedness of solutions. *Bol. Soc. Mat. Mexicana* (2) 5 (1960), 146-151.

20. Asymptotic behavior of solutions of non-autonomous system near sets. *J. Math. Kyoto Univ.* **1** (1961/1962), 303–323.

21. Asymptotic behavior of solutions of a system of differential equations. *Contributions to Differential Equations* **1** (1963), 371–387.

22. Stability of solutions of non-linear differential equations (in Japanese). *Sugaku* **13** (1961/1962), 228–236.

23. Stable sets and periodic solutions in a perturbed system. *Contributions to Differential Equations* **2** (1963), 407–420.

24. Stability of sets and perturbed system. *Funkcial. Ekvac.* **5** (1963), 31–69.

25. Extreme stability and almost periodic solutions of functional differential equations. *Arch. Rational Mech. Anal.* **17** (1964), 148–170.

26. Some notes on stability of sets and perturbed system. *Funkcial. Ekvac.* **6** (1964), 1–11.

27. Asymptotic stability of solutions of an almost periodic system of functional differential equations. *Rend. Circ. Mat. Palermo* (2) **13** (1964), 209–221.

28. Eventual properties and quasi-asymptotic stability of a non-compact set. *Funkcial. Ekvac.* **8** (1966), 79–90.

29. Stability Theory by Liapunov's Second Method. Publications of the Mathematical Society of Japan 9, Math. Soc. of Japan, Tokyo, 1966, viii+223 pp.

30. Linear system and its perturbed system. *Funkcial. Ekvac.* **9** (1966), 9–15. (with Junji Kato)

31. Stability and existence of a periodic solution. *J. Differential Equations* **4** (1968), 121–129.

32. Existence of a globally uniform-asymptotically stable periodic and almost periodic solution. *Tohoku Math. J.* (2) **19** (1967), 423–428.

33. Introduction to Ordinary Differential Equations (in Japanese). Asakura Shoten, Tokyo, 1967.

34. Asymptotically almost periodic solutions of an almost periodic system. *Funkcial. Ekvac.* **12** (1969), 23–40.

35. A relationship between uniformly asymptotic stability and total stability. *Funkcial. Ekvac.* **12** (1969/1970), 233–238. (with Junji Kato)

36. Oscillatory property of solutions of second order differential equations. *Tohoku Math. J.* (2) **22** (1970), 619–634.

37. Stability Theory and the Existence of Periodic Solutions and Almost Periodic Solutions. Applied Mathematical Sciences 14, Springer-Verlag, New York-Heidelberg, 1975, vii+233 pp.

38. Remarks on global properties in limiting equations. *Funkcial. Ekvac.* **24** (1981), 363–371. (with Junji Kato)

39. Attractivity in nonautonomous systems. *International J. Non-linear Mech.* **20** (1985), 519–528.

40. Total stability property in limiting equations for a functional-differential equation with infinite delay. *Casopis Pest. Mat.* **111** (1986), 62–69, 90. (with Yoshiyuki Hino)

41. Almost periodic solutions in an integrodifferential equation. *Proc. Roy. Soc. Edinburgh. Sect. A.* **114** (1990), 151–159. (with Yoshihiro Hamaya)

42. Relationships between BC-stabilities and p-stabilities and ρ-stabilities in functional-differential equations with infinite delay. *Thoku Math. J.* (2) **44** (1992), 45–57. (with Satoru Murakami)

43. Stability properties and the existence of an almost periodic solution in an integrodifferential equation. Delay and Differential Equations (Ames, I A, 1991), 1–8, World Sci. Publishing, River Edge, NJ, 1992.

44. Asymptotic behavior in a system of Volterra integrodifferential equations with diffusion. *Dynam. Systems Appl.* **3** (1994), 175–188. (with Satoru Murakami)

45. Existence of almost periodic solutions of some functional differential equations with infinite delay in a Banach space. *Tohoku Math. J.* (2) **49** (1997), 133–147. (with Yoshiyuki Hino and Satoru Murakami)