Nonlinear Dynamics and Systems Theory, **13** (4) (2013) 400–411

# Infinitely Many Solutions for a Discrete Fourth Order Boundary Value Problem

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Received: December 10, 2012; Revised: October 9, 2013

**Abstract:** By using variational methods and critical point theory, the authors obtain criteria for the existence of infinitely many solutions to the fourth order discrete boundary value problem

$$\begin{cases} \Delta^4 u(t-2) - \alpha\Delta^2 u(t-1) + \beta u(t) = \lambda f(t, u(t)), \quad t \in [1, T]_{\mathbb{Z}}, \\ u(0) = \Delta u(-1) = \Delta^2 u(T) = 0,\ \Delta^3 u(T-1) - \alpha\Delta u(T) = \mu g(u(T+1)), \end{cases}$$

where $T \geq 2$ is an integer, $[1, T]_{\mathbb{Z}} = \{1, 2, \dots, T\}$, $\alpha, \beta, \lambda, \mu \in \mathbb{R}$ are parameters, $f \in C([1, T]_{\mathbb{Z}} \times \mathbb{R}, \mathbb{R})$, and $g \in C(\mathbb{R}, \mathbb{R})$. Several consequences of their main theorems are also presented. One example is included to show the applicability of the results.

**Keywords:** *discrete boundary value problem; infinitely many solutions; fourth order; variational methods.*

**Mathematics Subject Classification (2010):** 39A10, 34B08, 34B15, 58E30.

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