Existence and Uniqueness of a Nontrivial Solution for Second Order Nonlinear $m$-Point Eigenvalue Problems on Time Scales

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Abstract: In this paper, by introducing a new operator, improving and generating a $p$-Laplace operator for some $p \geq 2$, we study the existence and uniqueness of a nontrivial solution for nonlinear $m$-point eigenvalue problems on time scales. We obtain several sufficient conditions of the existence and uniqueness of nontrivial solution of the eigenvalue problems when $\lambda$ is in some interval. Our approach is based on the Leray - Schauder nonlinear alternative.

Key Words: nontrivial solutions; eigenvalue problems; fixed point theorems; time scales.


1 Introduction

In this paper, we are concerned with the existence and uniqueness of a nontrivial solution for the following second order $m$-point eigenvalue problems on time scales:

\begin{align*}
(\varphi(h(t)u^\Delta(t)))^\Delta + \lambda f(t, u(t), u^\Delta(t)) &= 0, \quad t \in [0, T], \\
\alpha u(\rho(0)) - \beta u^\Delta(\rho(0)) &= C_0 \left( \sum_{i=1}^{m-2} \alpha_i u^\Delta(\xi_i) \right), \quad u^\Delta(T) = 0,
\end{align*}

where $\varphi: R \to R$ is an increasing homeomorphism and homomorphism such that $\varphi(0) = 0$, $\lambda > 0$ is a parameter, $\xi_i \in [0, T]$ with $0 < \xi_1 < \ldots < \xi_{m-2} < T$, $\alpha > 0$ and $\beta \geq 0$.

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