Nonlinear Dynamics and Systems Theory, **13** (4) (2013) 389–399

# Existence and Uniqueness of a Nontrivial Solution for Second Order Nonlinear $m$-Point Eigenvalue Problems on Time Scales

A. Denk and S. Topal *

*Department of Mathematics, Ege University,*
*35100 Bornova, Izmir-Turkey*

Received: November 1, 2012; Revised: October 21, 2013

**Abstract:** In this paper, by introducing a new operator, improving and generating a p-Laplace operator for some $p \geq 2$, we study the existence and uniqueness of a nontrivial solution for nonlinear m-point eigenvalue problems on time scales. We obtain several sufficient conditions of the existence and uniqueness of nontrivial solution of the eigenvalue problems when $\lambda$ is in some interval. Our approach is based on the Leray - Schauder nonlinear alternative.

**Keywords:** *nontrivial solutions; eigenvalue problems; fixed point theorems; time scales.*

**Mathematics Subject Classification (2010):** 34B15, 39A10.

## 1 Introduction

In this paper, we are concerned with the existence and uniqueness of a nontrivial solution for the following second order m-point eigenvalue problems on time scales:

$$(\varphi(h(t)u^{\triangle}(t)))^{\nabla} + \lambda f(t, u(t), u^{\triangle}(t)) = 0, \qquad t \in [0, T], \tag{1}$$

$$\alpha u(\rho(0)) - \beta u^{\triangle}(\rho(0)) = C_0(\sum_{i=1}^{m-2} \alpha_i u^{\triangle}(\xi_i)), \qquad u^{\triangle}(T) = 0, \tag{2}$$

where $\varphi : R \to R$ is an increasing homeomorphism and homomorphism such that $\varphi(0) = 0$, $\lambda > 0$ is a parameter, $\xi_i \in [0, T]$ with $0 < \xi_1 < ... < \xi_{m-2} < T$, $\alpha > 0$ and $\beta \geq 0$.

* Corresponding author: mailto:f.serap.topal@ege.edu.tr