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Design of an Optimal Stabilizing Control Law for Discrete-Time Nonlinear Systems Based on Passivity Characteristic

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Abstract: This paper proposes a passivity-based static output feedback law which stabilizes a broad class of nonlinear discrete time systems. This control law is designed in such a way that an arbitrary cost function is also minimized. A general structure with adjustable parameters is considered for the static feedback law. In order to find these parameters for solving the corresponding optimization problem, the genetic optimization algorithm is utilized. An illustrative example shows the effectiveness of the proposed approach.

Keywords: *nonlinear discrete-time systems; passivity-based control; optimal control; genetic optimization algorithm.*

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1 Introduction

The concept of passivity provides a useful tool for the analysis of nonlinear systems [1,2]. The main motivation for studying passivity in the system theory is its connection with stability [3–5]. A very important result in this field is the well known Kalman-Yakubovich-Popov (KYP) Lemma or Positive Real Lemma (PR) which has been specifically developed in the papers ([6, 7]). Also, Byrnes and Isidori [8] have shown that a number of stabilization theorems can be derived from the basic stability property of passive systems.

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The study of dissipative discrete-time systems, first was presented in [9] for linear systems. The motivation of studying dissipative discrete-time systems stems from the fact that passivity properties can simplify the system analysis. In [10, 11] nonlinear discrete-time systems which are affine in the control input, have been studied and some theorems on passivity-based control design were presented. Another approach to passivity in the nonlinear discrete-time case is presented by Monaco and Normand-Cyrot in [12, 13]. They obtained KYP conditions for single-input multiple-output general non-affine-in-input systems. Another problem treated is the action of making a system passive by means of a static state feedback, which is known as feedback passivity (passification). Sufficient conditions to convert MIMO non-passive systems to passive ones have been proposed in the series of papers [14]– [17].

The problem of stabilization is of high importance in the field of control. If a nonlinear discrete-time system is zero-state detectable and passive (with a positive definite and proper storage function) then the origin can be globally stabilized by $u = -\varphi(y)$, where φ is any locally Lipschitz function such that $\varphi(0) = 0$ and $y^T\varphi(y) > 0$ for all $y \neq 0$ ([11]). There is a great freedom in choosing the function $\varphi(y)$. The purpose of this paper is to use this freedom in such a manner that a given cost function be also minimized. Therefore, a general structure is considered for $\varphi(y)$ (which satisfies the above conditions) and adjustable coefficients in the proposed structure are found by a genetic optimization algorithm. This is worth noting that this idea can also be used for passive continuous-time systems.

The remainder of this paper is organized as follows. In the next section, the basic definitions and theorem about passive nonlinear discrete-time systems are presented. Section 3 presents the stabilizing controller design for passive systems in such a way that to minimize an appropriate cost function. A design example is given in Section 4. Finally, conclusions are presented in Section 5.

2 Preliminaries

In this section some basic definitions about the concept of passivity in the discrete-time systems are introduced.

A general class of discrete-time systems can be described by the nonlinear ordinary difference equation in the following discrete-time state space form:

$$\begin{aligned} x(k+1) &= F(x(k), u(k)), \\ y(k) &= H(x(k), u(k)), \end{aligned} \tag{1}$$

where $x \in D \subseteq R^n$ is the state vector, $u \in \mathrm{U} \subseteq R^m$ is the control input, and $y \in R^m$ is the system output. Suppose that F and H are both smooth mappings of the appropriate dimensions. Moreover, assume that $F(0, 0) = 0$ and $H(0, 0) = 0$. In this situation, a positive definite scalar function $V(x(k)) : D \to R$ (where $V(0) = 0$) is addressed as storage function and system (1) is said to be locally passive if there exists a storage function $V(x(k))$ such that:

$$V(F(x, u)) - V(x) \leq y^T u \qquad \forall (x, u) \in \mathrm{D} \times \mathrm{U}, \tag{2}$$

where $\mathrm{D} \times \mathrm{U}$ is a neighborhood of x=0, u=0.

Definition 2.1 [11] The zero dynamics of system (1) is defined by $F^* = F(x, u^*)$, where $(x, u^*) = \{(x, u) :\ \text{s.t. } H(x, u) = 0\}$. A system of the form (1) has a locally passive zero dynamics if there exists a positive definite function $V(x(k)) : D \to R$ such that:

$$V(F(x, u^*)) \leq V(x) \qquad \forall x \in D. \tag{3}$$

Definition 2.2 [11] A system (1) has local relative degree zero at x=0, if

$$\left. \frac{\partial H(x,u)}{\partial u} \right|_{\substack{x=0 \\ u=0}} \tag{4}$$

is nonsingular.

Now, assume that the nonlinear discrete-time system (1) is affine in the control input:

$$\begin{aligned} x(k+1) &= f(x(k)) + g(x(k))u(k), \\ y(k) &= h(x(k)) + J(x(k))u(k). \end{aligned} \tag{5}$$

The system (5) has local relative degree zero if $J(0)$ is nonsingular and it has uniform relative degree zero if $J(x)$ is nonsingular for all $x \in$ D. Additionally, the system (5) is locally zero-state observable if for all $x \in$ D,

$$y(k)|_{u(k)=0} = h(\phi(k, x, 0)) = 0 \quad \forall k \in Z^+ \Rightarrow x = 0, \tag{6}$$

where $\phi(k, x, 0) = f^k(x) = f(f^{k-1}(x)), \forall k > 1$, and $f^0(x) = x$. Also, $f^k(x)$ is the trajectory of the unforced dynamics $x(k+1) = f(x(k))$ from $x(0)$=x. If D $= R^n$, the system is globally zero-state observable. Moreover, system (5) is locally zero-state detectable if for all $x \in$ D, $y(k)|_{u(k)=0} = h(\phi(k, x, 0)) = 0$ and also for all $k \in Z^+$ implies $\lim_{k\to\infty} \phi(k, x, 0) = 0$. Also, if D $= R^n$, the system is globally zero-state detectable.

Another important asset of passive systems is their highly desirable stability properties which may simplify system analysis and controller design procedure. Therefore, transformation of a non-passive system into a passive one is desirable. The use of feedback to transform a non-passive system into a passive one is known as feedback passivation [12].

Definition 2.3 Let $\alpha(x)$ and $\beta(x)$ be smooth functions. Consider a static state feedback control law of the following form:

$$u(x) = \alpha(x) + \beta(x)w(k). \tag{7}$$

A feedback control law of the form (9) is regular if for all $x \in$ D, it follows that $\beta(x)$ is invertible. In order to analyze feedback passivation, the following theorem is taken from [16].

Theorem 2.1 *Consider a system of the form* (5). *Suppose* $h(0) = 0$ *and there exists a storage function V, which is positive definite* C^2 *function (i.e., the storage function and its first and second derivation is continuous). Also,* $V(0) = 0$ *and* $V(f(x) + g(x)u)$ *is quadratic in u. Then, system* (5) *is locally feedback equivalent to a passive system with V as storage function by a regular feedback control law of the form* (7) *if and only if the system has local relative degree zero at* $x = 0$ *and its zero dynamic is locally passive in a neighborhood of* $x = 0$.

Proof. See [11]. □

It is shown in [16] that control law of the form (7) with:

$$\alpha(x) = -J^{-1}(x)h(x) + J^{-1}(x)\bar{h}(x), \tag{8}$$

$$\beta(x) = J^{-1}(x)\bar{J}(x), \tag{9}$$

converts the non-passive nonlinear discrete-system (5) to a new passive dynamic given by:

$$\begin{aligned} x(k+1) &= f^*(x(k)) + g^*(x(k))\bar{h}(x(k)) + g^*(x(k))\bar{J}(x)w(k), \\ y(k) &= \bar{h}(x(k)) + \bar{J}(x)w(k), \end{aligned} \tag{10}$$

where

$$f^*(x) = f(x) - g(x)J^{-1}(x)h(x), \tag{11}$$

$$g^*(x) = g(x)J^{-1}(x), \tag{12}$$

$$\bar{J}(x) = (\frac{1}{2}g^{*T}\left.\frac{\partial^2 V}{\partial z^2}\right|_{z=f^*(x)} g^*(x))^{-1}, \tag{13}$$

$$\bar{h}(x) = -\bar{J}(x)(\left.\frac{\partial V}{\partial z}\right|_{z=f^*(x)} g^*(x))^{-1}. \tag{14}$$

3 Passivity-Based Optimal Control

Suppose that a system of the form (5) is passive with a positive definite storage function V. Let φ be any smooth mapping such that $\varphi(0) = 0$ and $y^T\varphi(y) > 0$ for all $y \neq 0$. The basic idea of passivity-based control is illustrated in the following theorem [11].

Theorem 3.1 *If system* (5) *is zero-state detectable and passive with storage function V which is proper on R^n, then the smooth output feedback control law* (15) *globally asymptotically stabilizes the equilibrium x=0,*

$$u = -\varphi(y), \quad u, y \in R^m. \tag{15}$$

Proof. See [11]. □

There is a freedom in selection of vector function $\varphi(y)$. In this paper, by use of this freedom we want to design $\varphi(y)$ such that in addition to globally asymptotically stabilizing of the nonlinear system (5) a given cost function be also minimized.

For this purpose, the following general structure for vector function $\varphi(y) = [\varphi_1(y_1), \varphi_2(y_2), ..., \varphi_m(y_m)]^T$ is assumed. It has the structure of a vector function belonging to the first-third quadrant sector, which $y^T\varphi(y) > 0$ for all $y \neq 0$ and also $\varphi(0) = 0$.

$$\varphi_i(y_i) = a_{i0}y_i + a_{i1}y_i^3 + ... + a_{il}y_i^{2l+1} \quad \text{for } i = 1, ..., m, \tag{16}$$

where $a_{i0}, a_{i1}, a_{i2}, ..., a_{il}$ belong to $R^+$and the suitable $l \in z^+ \geq 0$ may be set by the designer. The task is to find these unknown coefficients such that the proposed static output feedback minimizes an appropriate cost function in the form $I(k) = \sum_{\bar{k}=0}^{k} L(x(\bar{k}); u(\bar{k}))$.

In order to obtain the minimum value of the considered cost function, the optimization procedure based on the theory of genetic algorithms is used. The genetic algorithms constitute a class of search and optimization methods, which imitate the principles of natural evolution. A pseudo-code outline of genetic algorithms is shown below. The population of chromosomes at time t is represented by the time-dependent variable $P(t)$, with the initial population of random estimates $P(0)$ [18].

procedure GA
begin
t=0;
initialize $P(t) = P(0)$;
evaluate $P(t)$*;*
while not finished do
begin
t=t+1;
select $P(t)$ *from* P $(t$ -1);
reproduce pairs in $P(t)$ *by*
begin
crossover;
mutation;
reinsertion;
end
evaluate $P(t)$*;*
end
end

Therefore, by utilization of the GA optimization process, the best coefficients of the proposed structure (Equation (16)) of output feedback control law may be found in such a way that to minimize the given cost function. In the optimization process, the corresponding cost function is considered as the fitness function of genetic algorithm.

4 Design Example

Consider the following nonlinear discrete-time system:

$$\begin{array}{l} x_1(k+1) = (x_1^2(k) + x_2^2(k) + u(k))\cos(x_2(k)), \\ x_2(k+1) = (x_1^2(k) + x_2^2(k) + u(k))\sin(x_2(k)), \\ y(k) = (x_1^2(k) + x_2^2(k)) + \frac{1}{x_1^2(k)+x_2^2(k)-0.25}u(k). \end{array} \tag{17}$$

The system (17) is not passive. Considering $V = \frac{1}{2}(x_1^2(k)+x_2^2(k))$ as storage function, the system can be rendered passive by means of a static state feedback control law, due to the fact that $J(x(k)) = \frac{1}{x_1^2(k)+x_2^2(k)-0.25}$ is invertible and the zero dynamics of system (17) is passive [16]. Therefore, the passifying control scheme, i.e. $u = \alpha(x) + \beta(x)w$, proposed by equations (8) and (9) is applied to (17). The passiefied system has the conditions of Theorem 3.1. Consequently, it can be locally asymptotically stabilized by output feedback $w = -\varphi(y)$, where w is the new input of passified system. The goal is finding a proper function $\varphi(y)$ in order to minimize the following cost function of the passified system:

$$I = \frac{1}{2}\sum_{k=0}^{\infty}(w^2(k) + x(k)^T x(k) + y(k)^2).$$

The proposed optimization process has been done for three cases.

Case 1: Only first term of (16) is considered ($\varphi_1(y) = a_0 y$).

Case 2: First two terms of (16) are considered ($\varphi_2(y) = a_0y + a_1y^3$).
Case 3: First three terms of (16) are considered ($\varphi_3(y) = a_0y + a_1y^3 + a_2y^5$).
The nonlinear static functions, resulting from GA optimization procedure are:

$$\begin{aligned} \varphi_1(y) &= 0.02755y, \\ \varphi_2(y) &= 0.0245y + 0.0451y^3, \\ \varphi_3(y) &= 0.021413y + 0.0296y^3 + 0.0258y^5. \end{aligned} \tag{18}$$

The passified dynamic is simulated for the initial conditions x_0=[-1, +1] and the three different control inputs: $w = -\varphi_1(y)$, $w = -\varphi_2(y)$ and $w = -\varphi_3(y)$. Figures 1, 2 and 3, present the responses of output, first and second states of passified dynamic, respectively. The simulation results show that by regulating the adjustable coefficients in (16) a suitable performance may be achieved. Additionally, comparison of results is given in Table 1. As seen, considering more terms of (16) may lead to a better performance.

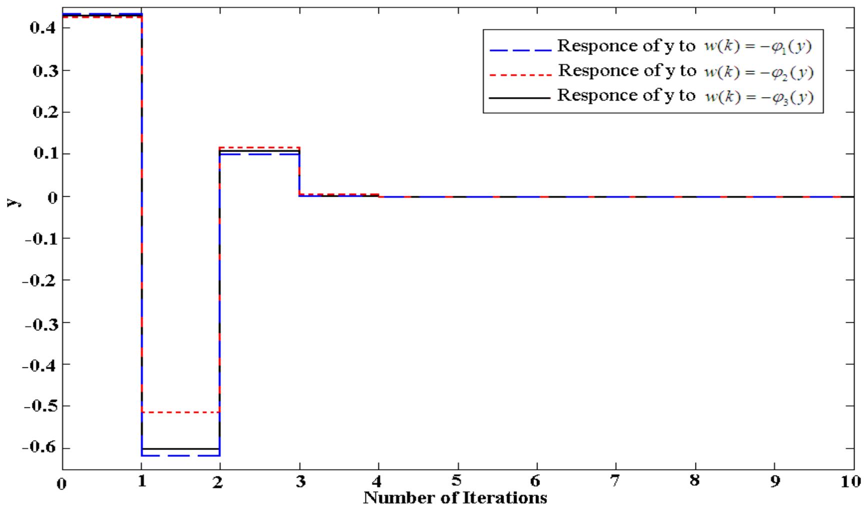

Figure 1: Time response of output $y(k)$.

	$w = -\varphi_1(y)$	$w = -\varphi_2(y)$	$w = -\varphi_3(y)$
$max\,\lvert y\rvert$	0.6178	0.5155	0.6028
I	2.619	2.6097	2.6027

Table 1: The cost functions (I) of control inputs, $w = -\varphi_1(y)$, $w = -\varphi_2(y)$ and $w = -\varphi_3(y)$.

5 Conclusion

In this paper, some properties of nonlinear discrete-time passive systems were studied. Based on the approach of passivity-based control, the output feedback u=-$\varphi(y)$, (where

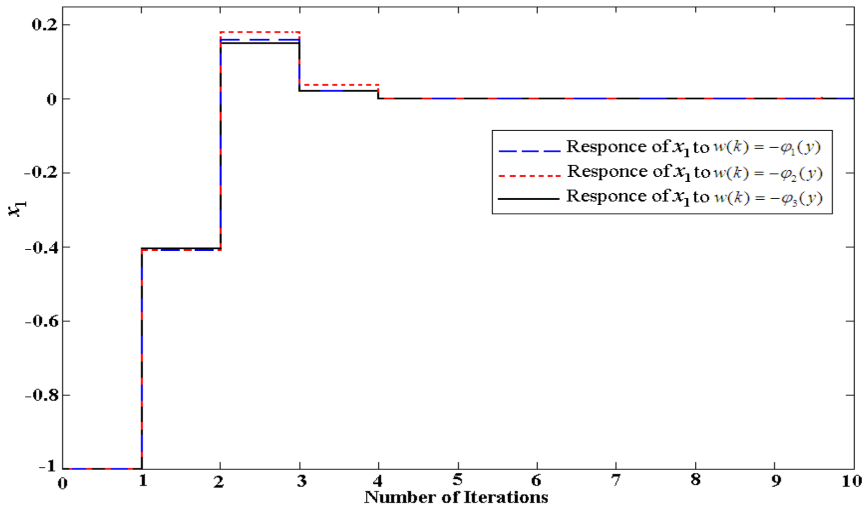

Figure 2: Time response of the first state $x_1(k)$.

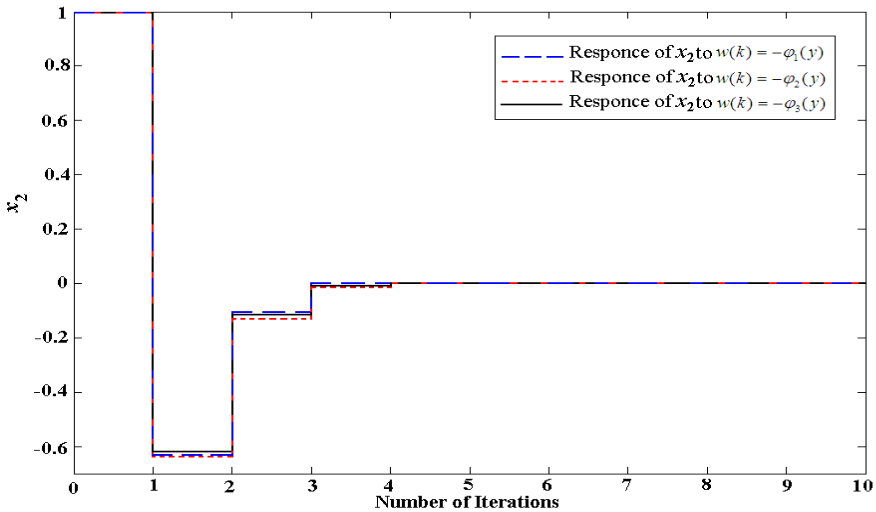

Figure 3: Time response of the second state $x_2(k)$.

$\varphi(y)$ is a smooth function belonging to a first-third quadrant sector) stabilizes the passive system. Having a freedom in choosing $\varphi(y)$, in addition to stabilization, one may consider optimization of the performance of the closed-loop system with respect to an appropriate index. Therefore, an extension (according to a general first-third quadrant sector function) with unknown coefficients was considered for $\varphi(y)$ and these coefficients were found based on the genetic optimization algorithm to minimize an appropriate cost function. Effectiveness of the proposed procedure was illustrated by an example.

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