Numerical Research of Periodic Solutions for a Class of Noncoercive Hamiltonian Systems

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Abstract: In this paper, we are interested in the existence of periodic solutions and approximative solutions to the Hamiltonian system \( \dot{x} = JH'(t, x) \) when \( H \) is non-coercive of the type \( H(t, r, p) = G(p - Ar) + h(t) \cdot (r, p) \). For the proof we use the Dual Action Principle and Critical Point Theory.

Keywords: Hamiltonian systems; periodic solutions; non-coercive; dual action principle; discrete dual action principle; critical point theory; numerical research.

Mathematics Subject Classification (2010): 34K28, 34K07, 34C25, 35A15.

1 Introduction

Let \( G : \mathbb{R}^n \to \mathbb{R} \) be a continuously differentiable function such that \( G' : \mathbb{R}^n \to G'(\mathbb{R}^n) \) be an homeomorphism. Let \( A \) be a matrix of order \( n \) and \( h : \mathbb{R} \to \mathbb{R}^n \) be a continuous \( T \)-periodic (\( T > 0 \)) function with zero mean value. Consider the non-coercive Hamiltonian

\[
H(t, r, p) = G(p - Ar) + h(t) \cdot (r, p).
\]

Here \( x, y \) is the usual inner product of \( x, y \in \mathbb{R}^{2n} \). We are interested in the boundary value problem

\[
\dot{x} = JH'(t, x) \quad (\mathcal{H})
\]

with

\[
x(0) = x(T). \quad (\mathcal{C})
\]

The goal of this work is to prove the existence of solutions to the problem \((\mathcal{H})(\mathcal{C})\) and to approximate these solutions.

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