



# Acceleration Control in Nonlinear Vibrating Systems Based on Damped Least Squares

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**Abstract:** A discrete time control algorithm using the damped least squares is introduced for acceleration and energy exchange controls in nonlinear vibrating systems. It is shown that the damping constant of least squares and sampling time step of the controller must be inversely related to insure that vanishing the time step has little effect on the results. The algorithm is illustrated on two linearly coupled Duffing oscillators near the 1:1 internal resonance. In particular, it is shown that varying the dissipation ratio of one of the two oscillators can significantly suppress the nonlinear beat phenomenon.

**Keywords:** *damped least squares; acceleration control.*

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## 1 Introduction

The damped least squares is a simple but effective analytical manipulation that helps to avoid singularity in practical minimization and control algorithms. It is also known as Levenberg-Marquardt method [11]. In order to illustrate the idea in simple terms, let us consider the minimization problem

$$\|E - A\delta u\|^2 \rightarrow \min, \quad (1)$$

where  $E \in R^n$  is a given vector, the notation  $\|\dots\|$  indicates the Euclidean norm in  $R^n$ ,  $A$  is typically a Jacobian matrix of  $n$  rows and  $m$  columns, and  $\delta u \in R^m$  is an unknown minimization vector. Although a formal solution of this problem is given by  $\delta u = (A^T A)^{-1} A^T E$ , the matrix product  $A^T A$  may appear to be singular so that no unique solution is possible. This fact usually points to multiple possibilities of achieving

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