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On Parameterized Lyapunov and Control Lyapunov Functions for Discrete-Time Systems

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Abstract: This paper deals with the existence and synthesis of parameterized-(control) Lyapunov functions (p-(C)LFs) for discrete-time nonlinear systems that are possibly subject to constraints. A p-LF is obtained by associating a finite set of parameters to a standard LF. A set-valued map, which generates admissible sets of parameters, is defined such that the corresponding p-LF enjoys standard Lyapunov properties. It is demonstrated that the so-obtained p-LFs offer non-conservative stability analysis conditions, even when Lyapunov functions with a particular structure, such as quadratic forms, are considered. Furthermore, possible methods for synthesizing p-CLFs are discussed. These methods require solving on-line a low-complexity convex optimization problem.

Keywords: *difference equations; asymptotic stability; Lyapunov methods; convex optimization.*

Mathematics Subject Classification (2010): 39A30, 37B25, 37L25.

1 Introduction

The problems considered in this paper are stability analysis and stabilizing controller synthesis via Lyapunov methods for discrete-time nonlinear systems that are possibly subject to constraints. It is well known that such methods rely on the existence and construction of a proper Lyapunov function (LF) [8, 11, 12, 19] and control Lyapunov function (CLF) [1, 9, 24], respectively. Unfortunately, the construction of LFs for general nonlinear systems is a very challenging problem. In particular, even linear systems with hard state/input constraints pose a non-trivial challenge to finding a non-conservative LF. As such, it would be desirable to identify a non-conservative class of Lyapunov

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