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On the Approximate Controllability of Fractional Order Control Systems with Delay

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Abstract: In this paper, sufficient conditions of the approximate controllability for a class of fractional order semilinear control systems with bounded delay are established. To illustrate the theory an example is given.

Keywords: *fractional order system; semilinear delay systems; mild solution; reachable set; approximate controllability.*

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1 Introduction

Let V and $\hat{V}$ be real Hilbert spaces. Also, let $Z = L_2([0,\tau];V)$ and $Y = L_2([0,\tau];\hat{V})$ be the corresponding function spaces defined on $[0,\tau]$. Let $C([-h,0],V)$ be the Banach space of all continuous functions from $[-h,0]$ to V with the supremum norm.

Consider the following fractional order semilinear control system with bounded delay

$$\left.\begin{array}{ll} {}^{C}D_t^{\alpha}x(t) = Ax(t) + Bu(t) + f(t,x_t), & t \in]0,\tau]; \\ x(t) = \varphi(t), & t \in [-h,0]. \end{array}\right\} \tag{1}$$

Here ${}^{C}D_t^{\alpha}$ is the Caputo fractional derivative of order α, where $1/2 < \alpha < 1$; the state $x(\cdot)$ takes its values in the space V; $A : D(A) \subseteq V \to V$ is a closed linear operator with dense domain $D(A)$ generating a C_0-semigroup $T(t)$; the control function $u(\cdot)$ takes its values in $\hat{V}$. The operator B is a bounded linear operator from $\hat{V}$ to V; $f : [0,\tau] \times C([-h,0],V) \to V$ is a continuous function and φ is the element of $C([-h,0];V)$.

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