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A Poiseuille Flow of an Incompressible Fluid with Nonconstant Viscosity

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Abstract: The viscosity coefficient in steady Navier-Stokes equations is determined for a particular velocity vector which arises from the study of conformal embedding of a Riemann surface.

Keywords: *Poiseuille flow; conformal embedding; nonconstant viscosity.*

Mathematics Subject Classification (2010): 76D06, 76M40, 35F10.

1 Introduction

Consider the steady Navier-Stokes equations

$$\begin{aligned} (\boldsymbol{u}\cdot\nabla)\boldsymbol{u} + \frac{1}{\rho}\nabla p &= \nu\Delta\boldsymbol{u} && \text{in } \Omega, \\ \nabla\cdot\boldsymbol{u} &= 0 && \text{in } \Omega, \\ \boldsymbol{u} &= \boldsymbol{0} && \text{on } \partial\Omega, \end{aligned}$$

in a long uniform tube $\Omega = \Omega_0 \times \mathbb{R}$ with the circular section

$$\Omega_0 := \{(x_1, x_2) \in \mathbb{R}^2;\ (x_1 - a)^2 + (x_2 - b)^2 < R^2\}.$$

Here, $\boldsymbol{u} = (u_1, u_2, u_3), p, \nu$ and ρ stand for the velocity field, the pressure, the viscosity and the density, respectively. We assume that ν and ρ are constant.

The solution of this problem with the additional assumption $u_1 = u_2 = 0$ is known as the Poiseuille flow. If this is the case, the pressure has a constant gradient $(0, 0, dp/dx_3)$ and u_3 is given by

$$u_3 = \frac{R^2 - (x_1 - a)^2 - (x_2 - b)^2}{4\nu\rho}\frac{dp}{dx_3}$$

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