Existence of Almost Periodic Solutions to Some Singular Differential Equations

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Abstract: In this paper we make use of the well-known Drazin inverse to study and obtain the existence of almost periodic solutions to some singular systems of first- and second-order differential equations with complex coefficients in the case when the forcing term is almost periodic. In order to illustrate our abstract results, an example will be discussed at the end of the paper.

Keywords: *Drazin inverse; almost periodic; singular system of differential equation; singular system of second-order differential equation.*

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1 Introduction

Let $\mathbb{C}^m$ be the m-dimensional complex space, which we equip with its natural Euclidean norm $|\cdot|$ and inner product $<\cdot,\cdot>$. Let $M(m,\mathbb{C})$ stand for the collection of all $m \times m$-square matrices with complex entries. If $A \in M(m,\mathbb{C})$ then its index which we will denote by $i(A)$ is the smallest nonnegative integer k such that

$$\mathrm{rank}(A^k) = \mathrm{rank}(A^{k+1}).$$

If $A \in M(m,\mathbb{C})$, then the Drazin inverse A^D of A is the matrix $X \in M(m,\mathbb{C})$ satisfying the following three properties:

$$AX = XA, \quad XAX = X, \quad XA^{k+1} = A^k,$$

where $k = i(A)$.

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