



# Approximate Controllability of Nonlocal Semilinear Time-varying Delay Control Systems

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**Abstract:** In this work the controllability problem for a class of semilinear control system with nonlocal initial conditions is considered. Under some simple conditions the relation between the reachable set of semilinear system and that of its corresponding linear system is established. In particular, approximate controllability of semilinear abstract control system is proved. Examples are presented to explain the application of the proposed result.

**Keywords:** *infinite-dimensional spaces; semilinear time-varying delay systems; approximate controllability; nonlocal conditions.*

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## 1 Introduction

Let  $(X, \|\cdot\|)$  be a Banach space and  $\mathcal{C}_t = C([-\tau, t]; X)$ ,  $\tau > 0$ ,  $0 \leq t \leq T < \infty$ , be a Banach space of all continuous functions from  $[-\tau, t]$  into  $X$  endowed with the norm  $\|\phi\|_{\mathcal{C}_t} = \sup_{-\tau \leq \eta \leq t} \|\phi(\eta)\|$ . Now, consider the following nonlocal semilinear delay control system

$$\begin{aligned}x'(t) &= Ax(t) + Bu(t) + f(t, x(t), x_{b(t)}) \text{ on } (0, T], \\h(x) &= \phi \text{ on } [-\tau, 0],\end{aligned}\tag{1}$$

where the state variable  $x(\cdot)$  takes values in Banach space  $X$  and the control function  $u(\cdot)$  belongs to  $Y = L^2([0, T]; U)$ , the Banach space of admissible control functions with a Banach space  $U$ . Standing assumptions on system operators are as follows:

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