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Partial Control Design for Nonlinear Control Systems

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Abstract: This paper presents a general approach to design a partially stabilizing controller for nonlinear systems. In this approach, the nonlinear control system is divided into two subsystems, which are called the *first* and the *second* subsystems. This division is done based on the required stability properties of system's states. Furthermore, it is shown that partial control makes the possibility of converting the control problem into a simpler one by reducing the number of control input variables. The reduced input vector (the vector that includes components of input vector appearing in the *first* subsystem) is designed based on the new introduced control Lyapunov function called *partial control Lyapunov function* (PCLF) to asymptotically stabilize the first subsystem.

Keywords: *partial stability; partial control; partial control Lyapunov function (PCLF).*

Mathematics Subject Classification (2010): 34D20, 37N35, 70K99, 74H55, 93C10, 93D15.

1 Introduction

The problem of partial stability, that is stability with respect to a part of system's states, finds applications in many of engineering problems. In particular, partial stability arises in the study of inertial navigation systems, spacecraft stabilization via gimbaled gyroscopes or flywheels, electromagnetic, adaptive stabilization, guidance, etc. [1]– [14]. In the mentioned applications, although the plant may be unstable (in the standard concept), it might be *partially* asymptotically stable, i.e., some states may have convergent behavior. It is in contrast to many other engineering problems where Lyapunov stability (in its standard concept) is required [17]– [20]. For example, consider the equation of motion for the reaction wheel pendulum depicted in Figure 1 [15]:

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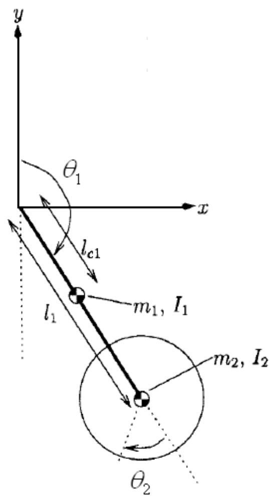

Figure 1: Coordinate convections for the reaction wheel pendulum [15].

$$\begin{aligned} & d_{11}\ddot{\theta}_1 + d_{12}\ddot{\theta}_2 + \phi\left(\theta_1\right) = 0, \\ & d_{21}\ddot{\theta}_1 + d_{22}\ddot{\theta}_2 = u, \end{aligned} \tag{1}$$

where θ_1 is the pendulum angle, θ_2 is the disk angle, u is the motor torque input and

$$\begin{aligned} & d_{11} = m_1 l_{c1}^2 + m_2 l_1^2 + I_1 + I_2, \\ & d_{12} = d_{21} = d_{22} = I_2, \\ & \phi(\theta_1) = -\bar{m} g \sin(\theta_1), \\ & \bar{m} = m_1 l_{c1} + m_2 l_1, \end{aligned} \tag{2}$$

where l_1 is the length of pendulum; l_{c1} is the position of the center of mass of the pendulum; m_1 is the mass of the pendulum; m_2 is the mass of disk; I_1, I_2 are the inertia of the pendulum and the disk around their center of masses. The reaction wheel pendulum is a physical pendulum with a symmetric disk attached to the end. The disk is free to spin about an axis which is parallel to the axis of rotation of the pendulum. Also, the disk is controlled by a DC-motor and the coupling torque generated by the angular acceleration of the disk can be used to actively control the system [15]. Suppose that a feedback control law should be designed so that $\dot{\theta}_1 \to 0$ and $\dot{\theta}_2$ be constant; that is, $\dot{\theta}_2\left(t\right) \to \Omega$ as $t \to \infty$ where $\Omega > 0$. This implies that $\theta_2\left(t\right) = \Omega t \to \infty$ as $t \to \infty$. Consequently, it is obvious that the reaction wheel pendulum is unstable in the standard concept; however, it is partially asymptotically stabilizable with respect to θ_1, $\dot{\theta}_1$ and $\dot{\theta}_2$.

Although partial stability has applications in many of engineering fields, there are a few papers regarding the design of control laws which stabilize only part of system's states [2]– [12] and advantages of partial control technique are not fully recognized. Among the existing papers in the field of partial control, most of them only consider a case study and try to design control laws for partial stability of their specific applications. Applications are Euler dynamical system [3], permanent rotations of a rigid body, relative equilibrium of a satellite, stationary motions of a gimbaled gyroscope [2] and chaos synchronization [7]. The references [2], [4], [9]– [11] focus on designing partial control and have given some way of designing. However, it is worth noting that the control schemes posed in these references are uneasy to realize and are usable only for systems with some special structures. In [12], a new class of nonlinear systems which is called "partially passive system" was introduced and some theorems for partial stabilization were developed.

In this paper, some new partial stabilization theorems for nonlinear dynamical systems are posed. It is shown that partial control makes the possibility of converting the control problem into a simpler one having fewer control input variables; which is one of the main contributions of this paper. In all of the existing papers in the field of partial control, the input vector is *wholly* designed; but in this paper by designing the reduced input vector, the advantage of partial control in simplifying the problem by reducing the control variables is recognized. The system's state is separated into two parts and accordingly the nonlinear dynamical system is divided into two subsystems. The subsystems, hereafter, are referred to as the "*first*" and the "*second*" subsystems. The reduced control input vector (the vector that includes components of input vector which appear in the *first* subsystem) is designed in such a way to guarantee asymptotic stability of the nonlinear system with respect to the first part of state vector. The design procedure is based on selection of a proper control Lyapunov function which is called partial control Lyapunov function. It's name is because that in this function only the first part of states is appeared.

The remainder of this paper is arranged as follows. First, the preliminaries on partial stability/control are given in Section 2. In Section 3, the theorems for partial control design are presented and explained in detail. Finally, conclusions are made in Section 4.

2 Preliminaries

In this section, the definitions and notations of partial stability are introduced. Consider a nonlinear system in the form;

$$\dot{x} = f(x), \qquad x(t_0) = x_0, \tag{3}$$

where $x \in R^n$ is the state vector. Let vectors x_1 and x_2 denote the partitions of the state vector, respectively. Therefore, $x = (x_1^T, x_2^T)^T$ where $x_1 \in R^{n_1}$, $x_2 \in R^{n_2}$ and $n_1 + n_2 = n$. As a result, the nonlinear system (3) can be divided into two parts (the *first* and the *second* subsystems) as follows

$$\begin{aligned} \dot{x}_1(t) &= F_1(x_1(t), x_2(t)), \qquad x_1(t_0) = x_{10}, \\ \dot{x}_2(t) &= F_2(x_1(t), x_2(t)), \qquad x_2(t_0) = x_{20}, \end{aligned} \tag{4}$$

where $x_1 \in D \subseteq R^{n_1}$, D is an open set including the origin, $x_2 \in R^{n_2}$ and $F_1 : D \times R^{n_2} \to R^{n_1}$ is such that for every $x_2 \in R^{n_2}$, $F_1(0, x_2) = 0$ and $F_1(., x_2)$ is locally Lipschitz in x_1. Also, $F_2 : D \times R^{n_2} \to R^{n_2}$ is such that for every $x_1 \in D$, $F_2(x_1, .)$ is locally Lipschitz in x_2, and $I_{x_0} = [0,\ \tau_{x_0})$, $0 < \tau_{x_0} \leq \infty$ is the maximal interval of existence of solution $(x_1(t), x_2(t))$ of (4) $\forall t \in I_{\mathrm{x}_0}$. Under these conditions, the existence and uniqueness of solution is ensured. Now, stability of the dynamical system (4) with respect to x_1 can be defined as follows [5]:

Definition 2.1 1. The nonlinear system (4) is Lyapunov stable with respect to x_1 if for every $\varepsilon > 0$ and $x_{20} \in R^{n_2}$, there exists $\delta(\varepsilon, x_{20}) > 0$ such that $\|x_{10}\| < \delta$ implies $\|x_1(t)\| < \varepsilon$ for all $t \geq 0$.

2. The nonlinear system (4) is asymptotically stable with respect to x_1, if it is Lyapunov stable with respect to x_1 and for every $x_{20} \in R^{n_2}$, there exists $\delta = \delta(x_{20}) > 0$ such that $\|x_{10}\| < \delta$ implies $\lim_{t\to\infty} x_1(t) = 0$.

It is important to note that this partial stability definition (which is given in [5]) is different from past definitions of partial stability [1,4]. In past definitions, it is required that $F_1(0,0) = 0$ and $F_2(0,0) = 0$. Also, the initial condition of the whole system should be in a neighborhood of the origin which is not required in Definition 2.1. The main advantage of considering the condition $F_1(0,x_2) = 0$ for every x_2, is that it makes the possibility of investigating the partial stability even if a part of system's states goes to infinity. Using this fact, authors of [5] present the unification of partial stability theory for autonomous systems and stability theory for nonlinear time-varying systems. This unification allows the stability theory of time-varying systems to be presented as a special case of autonomous partial stability theory.

In order to analyze partial stability, the following theorem and its corollary are taken from [5]. Note that in the following theorem, $\dot V(x_1,x_2) = V'(x_1,x_2)F(x_1,x_2)$ where the row vector of $\partial V(x)/\partial x$ is shown by $V'(x)$ and $F(x_1,x_2) = \begin{bmatrix} F_1^T(x_1,x_2) & F_2^T(x_1,x_2) \end{bmatrix}^T$.

Theorem 2.1 *Consider the nonlinear dynamical system (4). If there exist a continuously differentiable function $V : D \times R^{n_2} \to R$ and class K functions $\alpha(.)$ and $\gamma(.)$ such that*

$$V(0,x_2) = 0, \qquad x_2 \in R^{n_2}, \tag{5}$$

$$\alpha(\|x_1\|) \le V(x_1,x_2), \qquad (x_1,x_2) \in D \times R^{n_2}, \tag{6}$$

$$\dot V(x_1,x_2) \le -\gamma(\|x_1\|), \qquad (x_1,x_2) \in D \times R^{n_2}, \tag{7}$$

then, the nonlinear dynamical system (4) is asymptotically stable with respect to x_1.

Proof. See [5]. □

Corollary 2.1 *Consider the nonlinear dynamical system (4). If there exist a positive definite continuously differentiable function $V : D \to R$, and a class K function $\gamma(.)$ such that*

$$V'(x_1)F_1(x_1,x_2) \le -\gamma(\|x_1\|), \qquad (x_1,x_2) \in D \times R^{n_2}, \tag{8}$$

then, the equilibrium point of the nonlinear dynamical system (4) is asymptotically stable with respect to x_1.

Now, consider the following autonomous nonlinear control system:

$$\begin{aligned} \dot x_1(t) &= F_1(x_1,x_2,\mathrm{u}(x_1,x_2)), \qquad x_1(t_0) = x_{10}, \\ \dot x_2(t) &= F_2(x_1,x_2,\mathrm{u}(x_1,x_2)), \qquad x_2(t_0) = x_{20}, \end{aligned} \tag{9}$$

where $\mathrm{u} \in R^m$ and $F_1 : D \times R^{n_2} \times R^m \to R^{n_1}$ is such that for every $x_2 \in R^{n_2}$, $F_1(.,x_2,.)$ is locally Lipschitz in x_1 and u. Also, $F_2 : D \times R^{n_2} \times R^m \to R^{n_2}$ is such that for every $x_1 \in D$, $F_2(x_1,.,.)$ is locally Lipschitz in x_2 and u. These assumptions guarantee the local existence and uniqueness of the solution of the differential equations (9).

Definition 2.2 The nonlinear control system (9) is said to be asymptotically stabilizable with respect to x_1, if there exists some admissible feedback control law $\mathrm{u} = k(x_1,x_2)$, which makes system (9) asymptotically stable with respect to x_1.

3 An Approach for Partial Control Design

This section presents a feasible design algorithm for partial stabilization of nonlinear systems. Suppose the $\dot{x}_1$-subsystem in Eq. (9) is affine with respect to the control input (the $\dot{x}_2$-equation may have a general dynamical form). Therefore,

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + \textstyle\sum_{i=1}^{m} g_{1i}(x_1, x_2)u_i, \\ \dot{x}_2(t) &= F_2(x_1, x_2, \mathrm{u}), \end{aligned} \tag{10}$$

where u_i is the i^{th} component of input vector u. Also, $g_{1i} \in R^{n_1}$, for i=1,2,...,m. Let define r = number of $(g_{1i} \neq 0)_{i=1,\ldots,m}$. Hence, r is the number of components of input vector which appear in $\dot{x}_1$-subsystem. Thus $0 \leq \mathrm{r} \leq m$. Now, with respect to the value of r, two cases may be considered.

3.1 Case 1: $\mathrm{r} \neq 0$

By augmenting the r nonzero vectors g_{1i} in a matrix, the nonlinear system (10) can be rewritten as follows;

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2) + G_1(x_1, x_2)\mathrm{u}_1, \\ \dot{x}_2 &= F_2(x_1, x_2, \mathrm{u}), \end{aligned} \tag{11}$$

where $\mathrm{u}_1 \in R^{\mathrm{r}}$ is the reduced version of input vector u, that contains r control variables appearing in $\dot{x}_1$-subsystem, $G_1(x_1, x_2)$ is an $n_1 \times \mathrm{r}$ matrix where its columns are the r nonzero vectors g_{1i}. In this case, the task is to find an appropriate u_1, which guarantees partial stabilization of nonlinear system (11) with respect to x_1.

Theorem 3.1 *Consider the nonlinear dynamical system (11). Suppose $V(x_1) : D \to R$ is a positive definite continuously differentiable function (which is called partial control Lyapunov function) with the property that no solution x_1 of the unforced system* (11) *can stay identically in the set $\{V'(x_1) = 0\}$ other than the trivial solution $x_1(t) \equiv 0$. Also, suppose $\gamma(.)$ is class K function. Then, the system may be asymptotically stabilizable with respect to x_1 through the following reduced input vector*

$$\mathrm{u}_1 = k_1(x_1, x_2) = \begin{cases} \frac{b^T\{-V'(x_1)f_1 - \gamma(\|x_1\|)\}}{bb^T}, & where \;\; bb^T \neq 0, \\ 0, & where \;\; bb^T = 0, \end{cases} \tag{12}$$

where $b = V'(x_1)G_1(x_1, x_2)$. It is stressed that only in the points of state space $x_1 - x_2$ where $bb^T = 0$, the following condition should be satisfied:

$$V'(x_1)f_1(x_1, x_2) = -\gamma\left(\|x_1\|\right) \qquad \forall(x_1, x_2), \;\; \text{where}\; bb^T = 0. \tag{13}$$

Proof. By use of the control law (12), the time derivative of $V(x_1)$ in the line of system's trajectory is

$$\begin{aligned} \dot{V}(x_1) &= V'(x_1)\dot{x}_1 \\ &= V'(x_1)f_1 + V'(x_1)G_1\left[\frac{(V'(x_1)G_1)^T\{-V'(x_1)f_1 - \gamma(\|x_1\|)\}}{(V'(x_1)G_1)(V'(x_1)G_1)^T}\right] \\ &= V'(x_1)f_1 + \{-V'(x_1)f_1 - \gamma\left(\|x_1\|\right)\} \\ &= -\gamma\left(\|x_1\|\right). \end{aligned} \tag{14}$$

Therefore, according to Corollary 2.1, the nonlinear system (11) is asymptotically stable with respect to x_1. For the case where $bb^T = 0$, if condition (13) is satisfied, then by taking $\mathrm{u}_1 = 0$, partial stability will be achieved. □

Note: When $V'(x_1) = 0$, then $bb^T = 0$. In the points where $bb^T = 0$, condition (13) should be satisfied, which results in $\gamma(\|x_1\|) = 0$. Since, $\gamma(.)$ is a class K function, thus $\gamma(\|x_1\|) = 0 \Rightarrow x_1 = 0$. Therefore, as mentioned in Theorem 3.1, $V(x_1)$ should be chosen in a way that $V'(x_1) = 0 \Rightarrow x_1 \equiv 0$.

3.2 Case 2: r = 0

This situation means that there is no component of input vector in $\dot{x}_1$-subsystem. Suppose that $\dot{x}_2$-subsystem is affine with respect to input. Therefore,

$$\begin{aligned} \dot{x}_1 &= f_1(x_1, x_2), \\ \dot{x}_2 &= f_2(x_1, x_2) + G_2(x_1, x_2)\mathrm{u}. \end{aligned} \tag{15}$$

This system may be viewed as a cascade connection of two subsystems where x_2 is to be viewed as an input for *first* subsystem. The form (15) is usually referred to as the regular form. Assume that x_2 and u both belong to R^m (in other words, $n_2 = m$), and $G_2(x_1, x_2)$ is an m by m nonsingular matrix. This assumption is not so restrictive and many design methods, which are based on regular forms, e.g., backstepping or sliding mode techniques use such an assumption [16]. In this case, the task is to find an appropriate u; which guarantees partial stabilization of the closed-loop system.

Theorem 3.2 *Consider the nonlinear dynamical system (15). Suppose $V(x_1) : D \to R$ is a partial control Lyapunov function, $\gamma(.)$ is a class K function and $\varphi(x_1)$ is a smooth function. The design of the function $\varphi(x_1)$ is such that*

$$V'(x_1)(f_1(x_1, \varphi(x_1))) \le -\gamma(\|x_1\|). \tag{16}$$

Therefore, the nonlinear system (15) may be asymptotically stabilized with respect to x_1 by the following input vector

$$\mathrm{u} = G_2^{-1}[\varphi'(x_1) f_1 - f_2]. \tag{17}$$

Proof. Substitution of (17) in $\dot{x}_2$-subsystem (15) yields,

$$\begin{aligned} \dot{x}_2 &= f_2 + G_2\mathrm{u} \\ &= f_2 + G_2 G_2^{-1}[\varphi'(x_1) f_1 - f_2] \\ &= \varphi'(x_1) f_1 \end{aligned} \tag{18}$$

which results in $x_2 = \varphi(x_1)$. Since the condition (16) means that the *first* subsystem ($\dot{x}_1$-subsystem) may be asymptotically stabilized by a virtual input in the form $x_2 = \varphi(x_1)$ (according to Corollary 2.1). Therefore, the control law (17) partially stabilized the nonlinear system (15) with respect to x_1. □

3.3 Example. Partial stabilization of reaction wheel pendulum

The reaction wheel pendulum was described in Introduction. We define the states $z_1 = \theta_1$, $z_2 = \dot{\theta}_1$, $z_3 = \theta_2$ and $z_4 = \dot{\theta}_2$, The system's equations (1) can be written as follows

$$\begin{aligned} \dot{z}_1 &= z_2, \\ \dot{z}_2 &= -\tfrac{d_{22}}{\det D}\phi(z_1) - \tfrac{d_{12}}{\det D}u, \\ \dot{z}_3 &= z_4, \\ \dot{z}_4 &= \tfrac{d_{21}}{\det D}\phi(z_1) + \tfrac{d_{11}}{\det D}u, \end{aligned} \tag{19}$$

where $\det D = d_{11}d_{22} - d_{12}d_{21} > 0$. The problem is to stabilize the downward position of the pendulum, that is $z_1 = 0$, $z_2 = 0$, while stability of the rest of states is not of interest. Therefore, the state vector $x = [z_1, z_2, z_3, z_4]^T$ can be divided into $x_1 = [z_1, z_2]^T$ and $x_2 = [z_3, z_4]^T$. By separating the states into x_1 and x_2, one has: r = 1 and $u_1 = u$. The task is to design u according to Theorem 3.1 to achieve asymptotic stability with respect to x_1. Consider that for $\dot{x}_1$-subsystem $f_1 = \begin{bmatrix} z_2 & -\frac{d_{22}}{\det D}\phi(z_1) \end{bmatrix}^T$ and $G_1 = \begin{bmatrix} 0 & -\frac{d_{12}}{\det D} \end{bmatrix}^T$. By taking the partial control Lyapunov function $V(x_1) = 0.5\left(z_1^2 + z_1z_2 + z_2^2\right)$ then $b = V'(x_1)G_1 = -\frac{d_{12}}{\det D}(z_2 + 0.5z_1)$. Therefore, the points $bb^T = 0$ are equal to the points $z_2 = -0.5z_1$. First of all, the condition (13) should be checked. The left side of condition (13) is:

$$V'(x_1)f_1|_{bb^T=0} = V'(x_1)f_1|_{z_2=-0.5z_1} = -\frac{3}{8}z_1^2. \tag{20}$$

By choosing $\gamma(\|x_1\|) = \alpha z_1^2 + \beta z_2^2$; $\alpha,\beta > 0$, the positive constants α and β may be chosen such that $V'(x_1)f_1|_{z_2=-0.5z_1} = -\gamma(\|x_1\|)|_{z_2=-0.5z_1} = -\frac{3}{8}z_1^2$. This condition is satisfied for example for $\alpha = 0.25$ and $\beta = 0.5$. Now, according to Theorem 3.1, u is:

$$u = \begin{cases} -\frac{\det D}{d_{12}} \frac{-(z_1+0.5z_2)z_2 + \frac{d_{22}}{\det D}(z_2+0.5z_1)\phi(z_1) - 0.25z_1^2 - 0.5z_2^2}{z_2+0.5z_1} & for\ z_2 \neq -0.5z_1, \\ 0 & for\ z_2 = -0.5z_1. \end{cases} \tag{21}$$

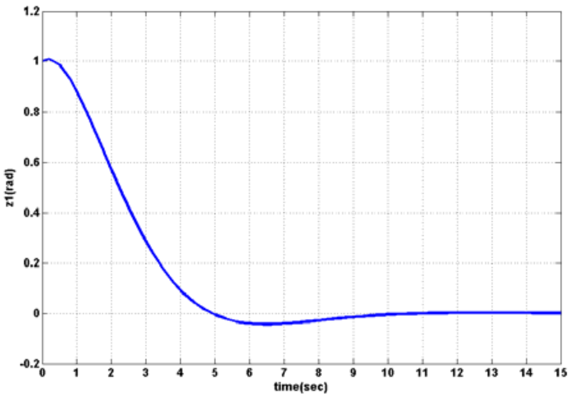

Figure 2: Time response of z_1 (the pendulum angle).

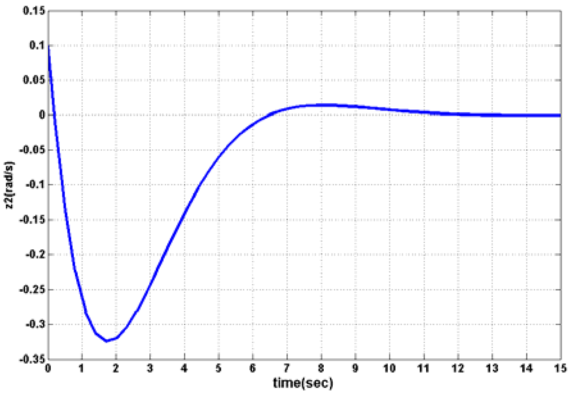

Fig gle).

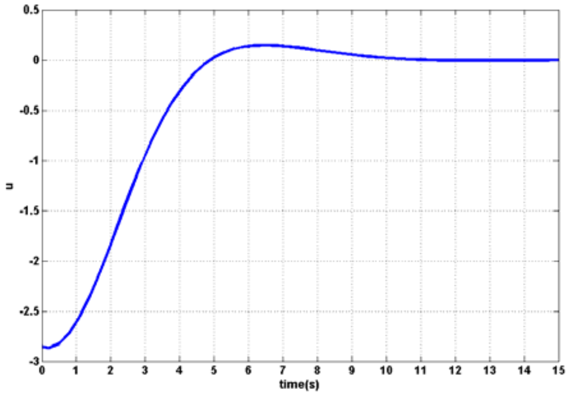

Figure 4: Time response of u (the motor torque input).

To check theoretical results, the closed loop system with controller (21) was simulated. The parameters of the system were chosen as $d_{11} = 0.004571$, $d_{22} = d_{12} = d_{21} = 2.495 \times 10^{-5}$, $\bar{m} = 0.35841$ that are physical parameters of the system located at the Automatic Control Dept., Lund Institute of Technology [15]. The initial conditions are

$$z_1(0) = 1, \quad z_2(0) = 0.1, \quad z_3(0) = z_4(0) = 0.$$

Figures 2 and 3 show the time responses of z_1 and z_2 in the closed loop system, respectively. As seen, the closed loop system shows quite fast convergence of z_1 and z_2 to zero. Also, the time response of controller (21) is shown in Figure 4.

4 Conclusion

In this paper, the problem of partial stabilization which has various applications in many of dynamic systems was considered and a general approach for stabilization of a nonlinear system with respect to a part of system's states was proposed. It was shown that in partial stabilization, the control input vector can be simplified by reducing its control variables. The reduced input vector was designed based on partial control Lyapunov function in a way that the asymptotic stabilization of a part of system's states was achieved. The proposed method was used in designing the partial controller for reaction wheel pendulum.

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