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Representation of the Solution for Linear System of Delay Equations with Distributed Parameters

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Abstract: The first boundary value problem for an autonomous system of linear delay partial differential equations of the second order has been solved. The solution is presented in an analytical form of formal series for the case, when matrices of coefficients are commutative and their eigenvalues are real and different. The obtained solution is studied on convergence and differentiability.

Keywords: *delay partial differential equation; first boundary value problem; time delay argument.*

Mathematics Subject Classification (2010): 35C10, 35G45, 35R10.

1 Introduction

Usually, when systems of differential equations are investigated, the main attention is paid to systems of ordinary differential equations (e.g., [1, 2]) or systems of partial differential equations [3]– [7]. Aside remains the analysis of systems of partial differential equations with delay. Their investigation is extremely rare [8]– [10].

Autonomous second-order systems of linear differential equations of with constant delay are considered in this paper:

$$\left\{ \begin{array}{l} \frac{\partial u(x,t)}{\partial t} = a_{11}\frac{\partial^2 u(x,t)}{\partial x^2} + a_{12}\frac{\partial^2 v(x,t)}{\partial x^2} + b_{11}u\left(x,t-\tau\right) + b_{12}v\left(x,t-\tau\right), \\ \frac{\partial v(x,t)}{\partial t} = a_{21}\frac{\partial^2 u(x,t)}{\partial x^2} + a_{22}\frac{\partial^2 v(x,t)}{\partial x^2} + b_{21}u\left(x,t-\tau\right) + b_{22}v\left(x,t-\tau\right). \end{array} \right. \tag{1}$$

We assume that matrices

$$A = \left[\begin{array}{cc} a_{11} & a_{12} \\ a_{21} & a_{22} \end{array} \right], B = \left[\begin{array}{cc} b_{11} & b_{12} \\ b_{21} & b_{22} \end{array} \right]$$

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