Positive Solutions for a Fourth Order Three Point Focal Boundary Value Problem

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Abstract: The authors consider a fourth order three point boundary value problem. Some a priori estimates to positive solutions for the boundary value problem are obtained. Sufficient conditions for the existence and nonexistence of positive solutions for the problem are established.

Keywords: fixed point theorem; cone; nonlinear boundary value problem; positive solution.

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1 Introduction

In this paper, we consider the fourth order differential equation

\begin{equation}
\frac{d^4}{dt^4}u(t) + g(t)f(u(t)) = 0, \quad 0 \leq t \leq 1,
\end{equation}

together with the boundary conditions

\begin{equation}
u(0) = u'(p) = u''(1) = u'''(1) = 0.
\end{equation}

Throughout this paper, we assume that

(H1) \( p \) is a real constant such that \( 1 - \sqrt{3}/3 \leq p \leq 1 \), \( f : [0, \infty) \to [0, \infty) \) and \( g : [0, 1] \to [0, \infty) \) are continuous functions, and \( g(t) \neq 0 \) on \([0, 1]\).

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