Nonlinear Dynamics and Systems Theory, **12** (2) (2012) 119–135

Time Scales Ostrowski and Grüss Type Inequalities Involving Three Functions

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Received: May 10, 2011; Revised: March 23, 2012

Abstract: In this paper, we present time scales versions of Ostrowski and Grüss type inequalities containing three functions. We assume that the second derivatives of these functions are bounded. Our results are new also for the discrete case.

Keywords: *Ostrowski–Grüss inequality; Ostrowski-like inequality; Montgomery identity; time scales.*

Mathematics Subject Classification (2010): 34N05, 26D15, 26D10.

1 Introduction

Motivated by a recent paper by B. G. Pachpatte [18], our purpose is to obtain time scales versions of some Ostrowski and Grüss type inequalities including three functions, whose second derivatives are bounded. In detail, we will prove time scales analogues of the following three theorems presented in [18].

Theorem 1.1 [See [18, Theorem 1]] *Let $f, g, h : [a, b] \to \mathbb{R}$ be twice differentiable functions on (a, b) such that $f'', g'', h'' : (a, b) \to \mathbb{R}$ are bounded, i.e.,*

$$\|f''\|_\infty := \sup_{t\in(a,b)} |f''(t)| < \infty, \quad \|g''\|_\infty < \infty, \quad \|h''\|_\infty < \infty.$$

Moreover, let

$$A[f, g, h] := gh \int_a^b f(s)\mathrm{d}s + fh \int_a^b g(s)\mathrm{d}s + fg \int_a^b h(s)\mathrm{d}s$$

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