Existence of Positive Solutions for the p-Laplacian with Nonlinear Boundary Conditions

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Abstract: In this paper, we consider a class of nonlinear elliptic problem with nonlinear boundary condition. The existence of positive solutions are established by sub-supersolution method and the Mountain Pass Lemma.

Keywords: *p-Laplacian equations; sub-supersolution; Mountain Pass Lemma; nonlinear boundary condition; positive solutions.*

Mathematics Subject Classification (2010): 35J55.

1 Introduction

In this paper, we are concerned with the following quasilinear elliptic problem

$$\begin{cases} -\Delta_p u + |u|^{p-2}u = f(x,u), & \text{in } \Omega, \\ |\nabla u|^{p-2}\frac{\partial u}{\partial \nu} = g(x,u), & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where $\Omega \subset \mathbb{R}^N (N \geq 3)$ is a bounded domain with smooth boundary $\partial\Omega$,
$\Delta_p u := \text{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian with $p > 1$ and $\frac{\partial}{\partial\nu}$ is the out normal derivative.

Recently, Afrouzi and Alizadeh [1] considered p-Laplacian equations with a nonlinear boundary condition, they developed a quasilinearization method in order to construct an iterative scheme that converges to a solution. They extended the results of [2] with $p \neq 2$. When $p = 2$, Song, Wang and Zhao [3] considered problem (1). By the sub-supersolution method, the existence of a positive solution was established. In [4], they

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