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Asymptotic Behavior of n-th Order Dynamic Equations

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Abstract: We are concerned with the asymptotic behavior of solutions of an n-th order linear dynamic equation on a time scale in terms of Taylor monomials. In particular, we describe the asymptotic behavior of the so-called (first) principal solution in terms of the Taylor monomial of degree $n-1$. Several interesting properties of the Taylor monomials are established so that we can prove our main results.

Keywords: *asymptotic behavior; dynamic equations; time scale; Taylor monomials, oscillation.*

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1 Introduction

We shall first consider the two term n-th order linear dynamic equation

$$u^{\Delta^n} + p(t)u(t) = 0, \quad p(t) > 0, \quad t \geq t_0 \tag{1}$$

on a time scale $\mathbb{T}$. Later (see Theorem 2.4) we consider a more general n-th order linear dynamic equation with $n+1$ terms. For the sake of completeness, we recall some basic definitions from the theory of time scales [7, 14].

A time scale $\mathbb{T}$ is an arbitrary nonempty closed subset of the real numbers. Since we are interested in oscillation results, we will consider time scales which are unbounded above, i.e., $\sup(\mathbb{T}) = \infty$. We use the notation $\mathbb{T} := [t_0, \infty)$.

For $t \in \mathbb{T}$ we define the forward and backward jump operators

$$\sigma(t) = \inf\{s \in \mathbb{T}, s > t\}, \quad \rho(t) = \sup\{s \in \mathbb{T}, s < t\}. \tag{2}$$

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