Nonlinear Dynamics and Systems Theory, **12** (1) (2012) 49–62

Existence and Uniqueness Conditions for a Class of $(k+4j)$-Point n-th Order Boundary Value Problems

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Received: April 14, 2011; Revised: December 20, 2011

Abstract: For the nth order nonlinear differential equation

$$y^{(n)} = f(x, y, y', \ldots, y^{(n-1)}),$$

we consider uniqueness implies uniqueness and existence results for solutions satisfying certain $(k+4j)-$point boundary conditions, $1 \le j \le n-1$ and $1 \le k \le n-2j$. We define $(j;k;j)-$point unique solvability in analogy to $k-$point disconjugacy and we show that $(j;n-2j;j)-$point unique solvability implies $(j;k;j)-$point unique solvability for $1 \le k \le n-2j$. This result is in analogy to $n-$point disconjugacy implies $k-$point disconjugacy, $2 \le k \le n-1$.

Keywords: *boundary value problem; uniqueness; existence; unique solvability; nonlinear interpolation.*

Mathematics Subject Classification (2010): 34B15, 34B10, 65D05.

1 Introduction

In this paper, we are concerned with uniqueness and existence of solutions for a class of boundary value problems for nth order ordinary differential equation, $n \ge 3$,

$$y^{(n)} = f(x, y, y', \ldots, y^{(n-1)}), \quad a < x < b, \tag{1}$$

subject to $n-2j$ conjugate boundary conditions and $2j$ nonlocal boundary conditions, where $j \ge 1$. In particular, given $1 \le k \le n-2j$, positive integers $m_1, \ldots, m_k$ such that $m_1 + \cdots + m_k = n-2j$, points $a < t_1 < \ldots < t_{2j} < x_1 < x_2 < \ldots < x_k < s_1 < \ldots < s_{2j} < b$

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