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Optimal State Observer Design for Nonlinear Dynamical Systems

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Abstract: This paper investigates the synthesis and the performance study of the optimal state observer designed for nonlinear dynamical systems to reconstruct the unmeasurable state variables and to stabilize rapidly the observation error system. The proposed nonlinear optimal state observer is based on the determination of the optimal observation gain matrix which is derived by minimizing a quadratic criterion formulated as an output feedback control problem of the observation error system. The gradient matrix operations is applied to the Lagrangian function in order to obtain necessary and sufficient conditions, for minimizing the proposed criterion, to perform the optimal gain matrix. The necessary and sufficient conditions are presented by coupled equations which resolution, by a numerical efficient algorithm, allows the calculus of the optimal observation gain. The effectiveness and the availability of the observer design approach are illustrated through numerical simulation to reconstruct the state variables of a robot with flexible link.

Keywords: *nonlinear observer design; optimal control; output feedback control; flexible robot.*

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1 Introduction

When the exact and complete knowledge on current states of a dynamic plant is impossible by different reasons, the use of a state observer (estimator) is compulsory to realize a successful closed-loop control [1–4].

Hence, the problem of state observation for nonlinear systems is of main importance in automatic control. In recent years many contributions have been presented in literature that investigate this problem for different classes of nonlinear systems. Generally, there are two approaches dealing with the nonlinear observer design. The first one is based on a nonlinear transformation by which the error dynamic is linear so that the design of state observer can be performed using linear techniques [5]. Necessary and sufficient conditions for the existence of the state transformation have been established in [5]. The second approach does not need any transformation and the observer design is directly based on the original system [2, 6].

For linear and nonlinear dynamical systems, a number of methods for observing the state variables and especially for the determination of the observation gain matrix, such that the asymptotic stability is ensured, have been proposed in the literature as the linear matrix inequality (LMI) approach [7–10], the Lyapunov equation method [11–13], the algebraic Riccati equation [5, 14, 15] and the min-max approach [16–19].

In synthesizing a control law and/or observation one two goals are focused: maximizing performances and minimizing costs of implementation. Hence, a simple control law, which is less complicated and less costly to implement than a full state feedback controller for example, may be preferred. Indeed, there is a number of structural alternatives such as full output feedback or low order dynamic compensation [20, 21].

In this paper we have considered the optimal state observer design for nonlinear dynamical systems which the non-linearity satisfy a globally Lipschitz condition. This approach is based on the minimizing of a quadratic criterion formulated as a quadratic output feedback control problem of the observation error in order to obtain an optimal gain. This proposed quadratic criterion has a direct signification and interpretation regarding to the desired observer. Thus, this optimal gain is calculated from the gradient resolution of the designed Lagrangain function in order to obtain necessary and sufficient conditions.

These necessary and sufficient conditions for the proposed nonlinear optimal state observer are derived, using the gradient techniques, in the form of Lyapunov and Riccati equations which resolution, by a proposed efficient iterative numerical algorithm, allows the calculus of the optimal gain matrix.

This paper is organized as follows: the proposed nonlinear optimal state observer is presented in Section 2. In Section 3, an illustrate example of a robot with flexible link is presented to highlight the performance of the proposed nonlinear optimal state observation approach.

2 Nonlinear Optimal State Observer

2.1 Problem formulation

We consider the class of nonlinear systems described by the following state equations

$$\begin{cases} \dot{x}(t) = Ax(t) + f\big(t, x, u\big), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^p$ is the control vector, $y(t) \in \mathbb{R}^m$ is the output vector, A and C are constant matrices of appropriate dimensions. The nonlinear fonction $f : \mathbb{R} \times \mathbb{R}^n \times \mathbb{R}^p \to \mathbb{R}^n$ is Lipschitz with respect to the state $x(t)$, uniformly in the control $u(t)$, that is, there exists a constant $\gamma > 0$ such that

$$\left\| f\big(t, x_1, u\big) - f\big(t, x_2, u\big) \right\| \leqslant \gamma \left\| x_1 - x_2 \right\| \tag{2}$$

for all $x_1(t), x_2(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^p$.

Systems with Lipschitz nonlinearity are common in many practical applications. Many nonlinear systems satisfy the Lipschitz property at least locally by representing them by a linear part plus a Lipschitz nonlinearity around their equilibrium points.

We assume that the pair (A, C) is observable. Then the state observer for the nonlinear system (1) may be written as follows

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + f\big(t, \hat{x}, u\big) + L\big(y(t) - \hat{y}(t)\big), \\ \hat{y}(t) = C\hat{x}(t), \end{cases} \tag{3}$$

with $\hat{x}(t) \in \mathbb{R}^n$ the state observer of $x(t)$ and $L \in \mathbb{R}^{n\times m}$ the observer gain matrix to be determined.

The observation error between the real state and the observed one is defined by

$$e(t) = x(t) - \hat{x}(t). \tag{4}$$

Subtracting (1) from (3) gives the dynamical reconstruction error

$$\dot{e}(t) = \big(A - LC\big)e(t) + f\big(t, x, u\big) - f\big(t, \hat{x}, u\big). \tag{5}$$

In literature, several methods can be used for the determination of the observer gain matrix, such that the asymptotic stability of the observation error is ensured, as the Lyapunov equation method, the algebraic Riccati equation approach and the linear matrix inequality (LMI) technique. The drawback of these methods is that the observation gain to determine can be practically not acceptable and where the minimization of a quadratic criterion is used has no direct physical interpretation regarding to the observation error dynamic.

In what follows we propose a new formulation of the dynamical observation error (5). Thus, the dynamical observation error can be considered as the following system

$$\begin{cases} \dot{e}(t) = Ae(t) + \eta(t) + f\big(t, x, u\big) - f\big(t, \hat{x}, u\big), \\ \eta(t) = -L\nu(t), \\ \nu(t) = Ce(t). \end{cases} \tag{6}$$

The system (6) expresses an output feedback control problem of the nonlinear system of order n with n dimensional input vector $\eta(t)$ and m dimensional output vector $\nu(t)$.

The proposed output feedback control problem scheme can be optimized by minimizing the following quadratic criterion defined by

$$\begin{aligned} J &= \int_0^\infty \Big(e^T(t) Q_0 e(t) + \eta^T(t) R_0 \eta(t) \Big) dt \\ &= \int_0^\infty e^T(t) \Big(Q_0 + C^T L^T R_0 L C \Big) e(t) dt \end{aligned} \tag{7}$$

with $Q_0 = Q_0^T \geq 0$ and $R_0 = R_0^T > 0$.

Then, we have the following result.

Theorem 2.1 *Consider the dynamical observation error (6). If there exists a matrix $P = P^T$ solution of the following algebraic Riccati equation*

$$(A - LC)^T P + P(A - LC) + \delta^{-1}P^2 + \delta\gamma^2 I + Q_0 + C^T L^T R_0 LC + Q = 0 \tag{8}$$

with $Q_0 = Q_0^T \geq 0$, $Q = Q^T \geq 0$, $R_0 = R_0^T$ and δ a positive scalar.

Then the state observation error is globally asymptotically stable and the quadratic criterion (7) satisfies

$$J \leq e_0^T P e_0, \tag{9}$$

where $e_0 = e(0)$ is the initial state observation error vector.

Proof In order to prove the asymptotic stability of the observation error (4), we consider the following quadratic Lyapunov function candidate

$$V(e(t)) = e(t)^T P e(t). \tag{10}$$

The observation error converges asymptotically towards zero if $V(e(t)) > 0$ and $\dot{V}(e(t)) < 0$ for all $e(t) \neq 0$.

The time derivative of $V(e(t))$ along any trajectory of (5) is given by

$$\begin{aligned}
\dot{V}(e(t)) &= \dot{e}^T(t)Pe(t) + e^T(t)P\dot{e}(t) \\
&= e^T(t)\Big[(A - LC)^T P + P(A - LC)\Big]e(t) + 2e^T(t)P\Big[f(t,x,u) - f(t,\hat{x},u)\Big] \\
&\leq e^T(t)\Big[(A - LC)^T P + P(A - LC) + \delta^{-1}P^2 + \delta\gamma^2 I\Big]e(t).
\end{aligned} \tag{11}$$

The inequality (11) is obtained by using the following relation

$$\begin{aligned}
2e^T(t)P\Big[f(t,x,u) - f(t,\hat{x},u)\Big] &\leqslant \delta^{-1}e^T(t)PPe(t) \\
&+ \delta\Big[f(t,x,u) - f(t,\hat{x},u)\Big]^T\Big[f(t,x,u) - f(t,\hat{x},u)\Big] \\
&\leqslant e^T(t)\Big[\delta^{-1}PP + \delta\gamma^2 I\Big]e(t).
\end{aligned}$$

The inequality (11) can be written as

$$\begin{aligned}
\dot{V}(e(t)) &\leq -e^T(t)Qe(t) - e^T(t)\Big(Q_0 + C^T L^T R_0 LC\Big)e(t) \\
&\leq -e^T(t)\Big(Q_0 + C^T L^T R_0 LC\Big)e(t) \\
&< 0,
\end{aligned} \tag{12}$$

where $(A - LC)^T P + P(A - LC) + \delta^{-1}PP + \delta\gamma^2 I + Q_0 + C^T L^T R_0 LC = -Q$.

Hence, $V(e(t))$ is a Lyapunov function for the system (5). Therefore, the observation error (4) is asymptotically stable. Furthermore, by integrating both sides of the inequality (12) from 0 to T and using the initial conditions, we have

$$V\left(e(T)\right) - V\left(e(0)\right) < -\int_0^T e^T(t)\Big(Q_0 + C^T L^T R_0 LC\Big)e(t)dt.$$

Since the system (4) is asymptotically stable, that is, $e(T) \to 0$, when $T \to \infty$, we obtain $V\left(e(T)\right) \to 0$. Thus we get

$$\begin{aligned} J &= \int_0^T e^T(t)\Big(Q_0 + C^T L^T R_0 LC\Big)e(t)dt \\ &< V(e(0)) \\ &< e_0^T P e_0. \end{aligned}$$

The proof of Theorem 2.1 is completed. □

At this stage, (6) and (9) form an optimization problem which, given an e_0, can be solved in order to obtain an optimal observation gain L for the nonlinear system. Unfortunately, this optimal gain L will in general depend on e_0. Thus, it would not really be a feedback control. In order to find an optimal observation gain that is independent of the initial observation error, it is necessary to overcome this problem. Then, we attempt to determine the optimal gain L in an average sense, if we view the initial observation error e_0 as a random variable uniformly distributed over the surface of an n dimensional unit sphere, it follows that

$$E\big\{e_0 e_0^T\big\} = I. \tag{13}$$

Then, the expected value of the quadratic criterion $\bar{J}$ of the cost function (9) is simply evaluated as follows

$$\bar{J} = E\big\{J\big\} \le E\big\{e_0^T P e_0\big\} = trace\big\{P\big\}. \tag{14}$$

Thus, that may have appeared to be a dynamical problem (5) is formulated as a static quadratic criterion (14) which is minimized with respect to the observation gain matrix L and the symmetric positive definite matrix P subject to the constraint (8).

2.2 Gain matrix optimization

The optimal observation gain matrix of the state observation (3), which ensures the asymptotic convergence of the state observation error (4), is given by the following theorem

Theorem 2.2 *We consider Theorem 2.1 and if there exists a matrix $\Gamma = \Gamma^T \ge 0$ solution of the Lyapunov equation*

$$\left(A - LC\right)\Gamma + \Gamma\left(A - LC\right)^T + 2\delta^{-1}P + I = 0. \tag{15}$$

Then the optimal observation gain matrix of the system (3) is given by

$$L = R_0^{-1} P \Gamma C^T (C \Gamma C^T)^{-1}. \tag{16}$$

Proof To obtain an optimality condition, define the corresponding Lagrange function as

$$\begin{aligned}\Im\big(L,P,\Gamma\big) \;&=\; trace\Big\{P\Big\}+trace\Big\{\Gamma^T\Big[\big(A-LC\big)^T P+P\big(A-LC\big)\\ &+\;\delta^{-1}PP+\delta\gamma^2 I+Q_0+C^T L^T R_0 LC+Q\Big]\Big\},\end{aligned} \tag{17}$$

where $\Gamma\in\mathbb{R}^{n\times n}$ is a matrix of Lagrangian multiplier may be selected symmetric positive definite.

To continue the developments, the following lemma is used.

Lemma 2.1 *For any matrices X,Y,A and B with appropriate dimensions, we have [22, 23]*

$$\frac{\partial}{\partial Y}trace\left\{X^TY\right\}=X,\quad \frac{\partial}{\partial Y}trace\left\{X^T\left(A+YB\right)\right\}=XB^T.$$

By using gradient matrix operations defined by Lemma 2.1 the necessary conditions for L,P and Γ to be optimal are given by

$$\frac{\partial\Im}{\partial L}\left(L,P,\Gamma\right)=-2P\Gamma C^T+2R_0LC\Gamma C^T=0, \tag{18}$$

$$\frac{\partial\Im}{\partial P}\left(L,P,\Gamma\right)=\left(A-LC\right)\Gamma+\Gamma\left(A-LC\right)^T+2\delta^{-1}P+I=0, \tag{19}$$

$$\begin{aligned}\frac{\partial\Im}{\partial \Gamma}\left(L,P,\Gamma\right)\;&=\;\left(A-LC\right)^T P+P\left(A-LC\right)+\delta^{-1}PP\\ &\qquad+\delta\gamma^2I+Q_0+C^TL^TR_0LC+Q=0.\end{aligned} \tag{20}$$

From equation (18), we obtain the optimal observation gain matrix L given by equation (16). □

In view of this, the last relations can be written to the following

$$\left\{\begin{array}{l} F_1\left(L,P,\Gamma\right):L=R_0^{-1}P\Gamma C^T\left(C\Gamma C^T\right)^{-1},\\ F_2\left(L,P,\Gamma\right):\left(A-LC\right)\Gamma+\Gamma\left(A-LC\right)^T+2\delta^{-1}P+I=0,\\ F_3\left(L,P,\Gamma\right):\left(A-LC\right)^TP+P\left(A-LC\right)+\delta^{-1}PP+\delta\gamma^2I\\ \qquad\qquad\qquad\qquad\qquad\qquad+Q_0+C^TL^TR_0LC+Q=0.\end{array}\right. \tag{21}$$

It is clear that the three equations of the system (21) are coupled. Then, to solve this system, it is important to propose the following iterative algorithm.

Algorithm 2.1 1. Initialize : Set $n=1$:

Select $Q_0\geq 0$, $Q\geq 0$, $R_0>0$ and L_1 such as $A-L_1C$ is stable.

2. n^{th} iteration:

- Using this value of L_n and the resolution of the algebraic Riccati equation $F_3(L_n,P_n)=0$, we obtain the value for P_n.
- With L_n,P_n and the resolution of the Lyapunov equation $F_2(L_n,P_n,\Gamma_n)=0$, we get Γ_n.

- Update L_{n+1}, for the obtained values P_n and Γ_n, with the relation $F_1(L_{n+1}, P_n, \Gamma_n)$.

3. $n = n + 1$:

   Repeat the step 2 for $n = n + 1$ to obtain the optimal values.

4. Terminate :

   Stop the algorithm if $\|P_n - P_{n-1}\| \leq \varepsilon$ (ε is a prescribed small number used to check the convergence of the algorithm).

   So, for $n = 1, 2, ...$, we have

   P_n is found from the Riccati equation $F_3(L_n, P_n) = 0$,

   Γ_n is found from the Lyapunov equation $F_2(L_n, P_n, \Gamma_n) = 0$,

   L_{n+1} is found from $F_1(L_{n+1}, P_n, \Gamma_n)$.

3 Numerical Example

To illustrate the availability and the efficiency of the proposed nonlinear optimal state observer design, we consider the system of a single link robot with a revolute elastic joint rotating in a vertical plane which is modelled by [8, 24]:

$$\begin{cases} \dot{\theta}_m = \omega_m, \\ \dot{\omega}_m = -\dfrac{F_m}{J_m}\omega_m + \dfrac{K}{J_m}(\theta_l - \theta_m) + \dfrac{K_\tau}{J_m}u, \\ \dot{\theta}_l = \omega_l, \\ \dot{\omega}_l = -\dfrac{F_l}{J_l}\omega_l - \dfrac{K}{J_l}(\theta_l - \theta_m) - \dfrac{Mgh}{J_l}\sin(\theta_l), \end{cases} \tag{22}$$

where θ_m, ω_m, θ_l and ω_l are the motor angular displacement, the angular velocity of the motor, the link angular displacement and the angular velocity of the link respectively. J_m and J_l are the inertia of the motor and link respectively, $2h$ and M represent the length and mass of the link, F_m and F_l are the viscous friction coefficients, K is the elastic constant, g is the gravity constant and K_τ is the amplifier gain. The control u is the torque delivered by the motor.

The system (22) can be rewritten under the form (1) in the following state representation

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{K}{J_m} & -\dfrac{F_m}{J_m} & \dfrac{K}{J_m} & 0 \\ 0 & 0 & 0 & 1 \\ \dfrac{K}{J_l} & 0 & -\dfrac{K}{J_l} & -\dfrac{F_l}{J_l} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \dfrac{K_\tau}{J_m} \\ 0 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0 \\ 0 \\ 0 \\ -\dfrac{ghM}{J_l} \end{bmatrix} \sin(x_3), \\ y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}, \end{cases} \tag{23}$$

with $\begin{bmatrix} x_1 & x_2 & x_3 & x_4 \end{bmatrix}^T = \begin{bmatrix} \theta_m & \omega_m & \theta_l & \omega_l \end{bmatrix}^T$.

The performances of the proposed nonlinear optimal state observer with the optimal gain obtained by the proposed iterative algorithm was investigated by simulation for the flexible link robot (23) characterized by the following numerical parameters (table 1) [8]:

Parameter	Numerical value
K	$1.8 Nm/rad$
K_τ	$0.8 Nm/V$
J_m	$37.9 \times 10^{-3} Kgm^2$
J_l	$94.6 \times 10^{-3} Kgm^2$
h	$0.15m$
M	$0.21 Kg$
F_m	$47.3 \times 10^{-3} Nm/rad/s$
F_l	$0 Nm/rad/s$

Table 1: Numerical parameters of the flexible link robot.

In the following, the procedure for the nonlinear optimal state observer design is presented. For the computation of the observation gain matrix L, we select the parameters $Q_0 = 0.75 \cdot I_4, Q = 0.75 \cdot I_4, R_0 = I_4$ and $\delta = 0.25$.

Using the proposed iterative algorithm described above for the given Q_0, Q, R_0 and δ the observation gain matrix can be found using MATLAB. If the results are not satisfactory, Q_0 and R_0 are modified and the procedure is repeated. After some design repetition and with the selected parameters, the outcomes of the iterative algorithm resolution after $N = 27$ iterations are the following:

• the optimal observation gain matrix:

$$L_{opt} = \begin{bmatrix} -11.6291 & 10.1573 \\ 9.8738 & -0.4123 \\ 39.1030 & -11.1101 \\ -5.3396 & 5.9703 \end{bmatrix},$$

• the symmetric positive definite matrix:

$$P_{opt} = \begin{bmatrix} 4.8652 & -0.2200 & -1.4939 & 0.9748 \\ -0.2200 & 0.9352 & -0.0126 & 0.1866 \\ -1.4939 & -0.0126 & 14.8564 & -4.2233 \\ 0.9748 & 0.1866 & -4.2233 & 2.7364 \end{bmatrix},$$

• the matrix of Lagrangian multiplier:

$$\Gamma_{opt} = \begin{bmatrix} 7.9319 & -4.2409 & 2.2174 & -2.1123 \\ -4.2409 & 6.2424 & -2.3009 & 2.3777 \\ 2.2174 & -2.3009 & 3.6516 & -1.8314 \\ -2.1123 & 2.3777 & -1.8314 & 1.6209 \end{bmatrix}.$$

The performances of the proposed nonlinear optimal state observer, tested by numerical simulation, are shown in Figures 1 to 4 which depict the evolution of the actual and the observed state variables of the studied flexible link robot: the motor angular position

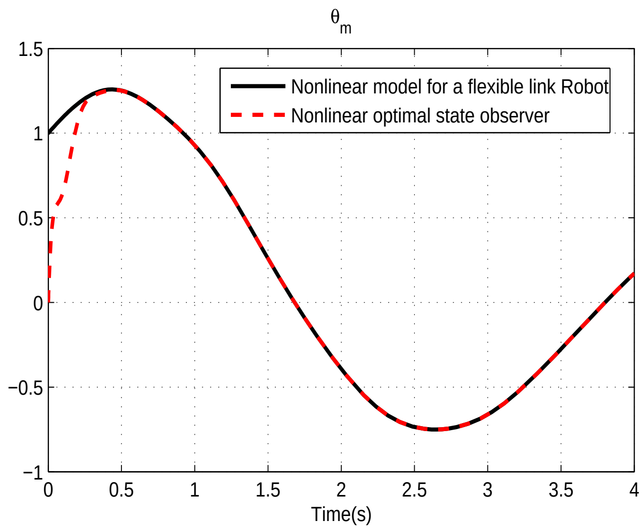

Figure 1: Actual and observed angular position θ_m of the motor.

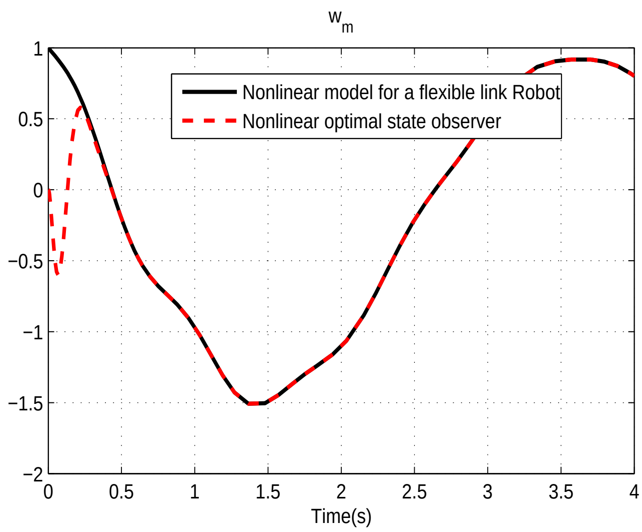

Figure 2: Actual and observed angular velocity ω_m of the motor.

θ_m, the motor angular velocity ω_m, the link angular position θ_l and the link angular velocity ω_l.

It appears, from these simulations, that the nonlinear optimal state observation approach allows a well reconstruction of the actual states. It can converge rapidly towards the state variable of the flexible link robot. Indeed, the high performances of the proposed nonlinear optimal state observer show the improvement led by the use of the proposed iterative algorithm permitting the calculus of the optimal gain matrix.

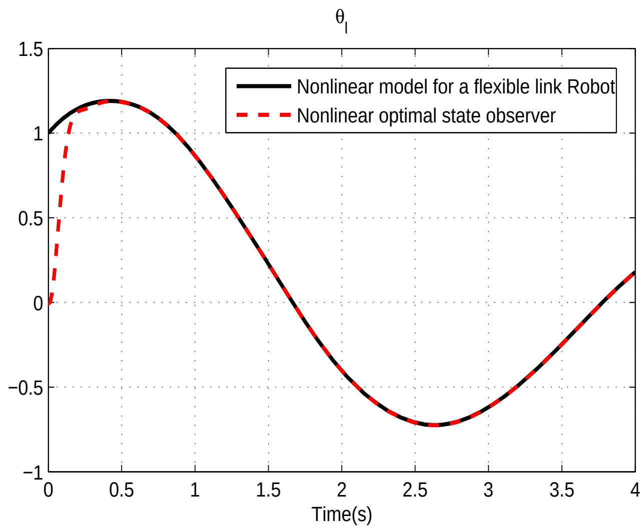

Figure 3: Actual and observed angular position θ_l of the link.

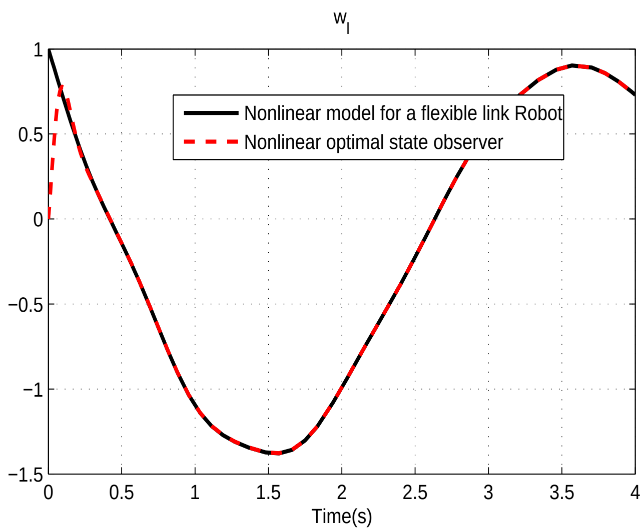

Figure 4: Actual and observed angular velocity ω_l of the link.

4 Conclusion

Nonlinear optimal state observer design for a class of continuous-time nonlinear systems, where the nonlinearity satisfy the Lipschitz condition, has been studied in this paper. The nonlinear optimal state observer is based on the determination of an optimal observing gain matrix derived by minimizing a quadratic criterion characterized by a quadratic output feedback control problem of the observation error system.

It has been shown from the simulation results that the proposed nonlinear optimal state observer allows the reconstruction of the unmeasurable state variables of the flexible link robot. Indeed, the performance improvement of the nonlinear optimal state observer is due to the design of a numerical efficient algorithm leading to the calculus of the optimal gain matrix.

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