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State Feedback Controller of Robinson Nuclear Plant with States and Control Constraints

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Abstract: This paper deals with the problem of finding a stabilizing feedback controller for nuclear reactor power plant. A mathematical model of the H. B. Robinson pressurized water reactor plant is formulated. The model includes representations for point kinetics, core heat transfer, piping, pressurizer, and the steam generator. The designed linear state feedback controller accounts for constraints on neutron flux level, steam pressure in steam generator, hot leg temperature and constraints on control inputs of reactivity and electric heater to pressurizer. Simulation results show the effectiveness of the proposed design.

Keywords: *H.B. Robinson nuclear plant; stabilization; state feedback controller; state constraints.*

Mathematics Subject Classification (2010): Primary: 34D20, 47H07; Secondary: 34C12, 47A50.

1 Introduction

Currently, there are more than 80 pressurized water reactors (PWRs) operating as important contributors to electricity supply worldwide. But, in this type of reactor, safety margins obstruct the optimal exploitation of the plant because instability may occur under particular operating conditions. The stability of PWR reactor systems has been of a great concern from the safety and the design point of view [1].

Stability problems may only arise during start up or during transients which significantly shift the operating point. Instructions for PWRs contain clear rules on how to

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avoid operating points (regions) that may produce power-void oscillations. The current trend of increasing reactor powers and of applying natural circulation core cooling, however, has major consequences for the stability of new PWR designs. These modifications have allowed PWRs to work at high nominal power, but they have also favored an increase in the reactivity feedback and a decrease in the response time, resulting in a lower stability margin when the reactor is operated at low mass flow and high nominal power [2]. The objective of improved control is to obtain higher plant productivity. Increasing 1) the plant availability, 2) the economic utilization of the nuclear fuel, and 3) the operational flexibility.

A new intelligent nonlinear control for power system stabilizers that improves the transient stability.This permits the most possible simple design implementation of an adaptive-fuzzy logic passivity-based controller which is developed on power system obtained by a suitable use of the backstepping technique [19]. It is difficult to overstate the importance of considering control constraints in control system design: such constraints have well-known implications for the behaviour of the resulting closed-loop system, and ignoring these constraints can lead to a dramatic loss of performance and, potentially, stability. Hassan and Boukas [20] show that the problem of stabilizing a linear quadratic regulator is subject to constraints on the state and the input vectors, Our technique relays on an iterative approach that uses the solution of the standard linear quadratic regulator as an initial guess for the optimal solution and then iteratively, the solution is improved by designing a controller that compensates for the violation of the constraints at each iteration .

Recently, several controller design techniques for constrained linear systems have been proposed. We provide a critical review of constraint compensation techniques for control systems with an emphasis on methods which have been successfully applied to process control problems. Most of these methods can be classified as: (i) anti-windup techniques; (ii) model predictive control techniques; and (iii) hybrid feedback linearization/model predictive control techniques. Anti-windup methods usually are based on applying linear anti-windup compensation to the linear system obtained from feedback linearization [12–16]. Model predictive control provides a very convenient framework for the control of constrained systems as input and output constraints can be incorporated directly into the associated controller [13–17]. Hybrid feedback linearization/model predictive control techniques utilize feedback linearization to generate a constrained linear system which is regulated with a linear model predictive controller [14–18].

Many approaches demonstrate the design of a robust controller using the linear quadratic gaussian with loop transfer recovery (LQG/ LTR) for nuclear reactors with the objective of keeping a desirable performance for reactor fuel temperature and temperature of the coolant leaving the reactor for a wide range of reactor power [15].

This paper deals with the problem of designing a stabilizing feedback controller for continuous H. B. Robinson pressurized water reactor plant which is in the form of linear state-variable model, where the control inputs (reactivity and electric heater to pressurizer) act additively. The model is based on mass, and energy balance; design data from the safety analysis report are used to evaluate the necessary coefficients. The model includes representations for point kinetics equations (six delayed neutron groups), core heat transfer, piping, pressurizer, and the steam generator [3].

The H. B. Robinson Nuclear Plant produces 2200 MW at full power. It includes a pressurized water reactor (PWR), pressurizer, and three vertical U-tube recirculation-type steam generators [3]. The practicality of the control schemes is demonstrated on the problem of finding a stabilizing controller for continuous H. B. Robinson pressurized water reactor plant subject to both state and control constraints.

Meanwhile the problem of stabilization with state and input constraints has been solved recently [4–6, 7–9]. Saberi [5] generalized Kaliora's result to a general linear system. Diao [6] constructed a semi-global stabilizing controller subject to both amplitude and rate constraints. Lin [10] constructed a semi global stabilizing controller subject to both amplitude and rate constraints. Castelan et al. [7] showed that the problem of designing a state feedback controller to constrain linear system $\dot{x} = Ax + Bu$ to a symmetric state constraint set $S = \{-w \prec Gx \prec w\}$ is solvable if rank G is less than or equal to the number of controls and the null space of G; ker G is A, B invariant [11]. Thus there exists an F such that ker G is $A + BF$ invariant, the eigenvalues of $(A + BF)_{kerG}$ are in the open left-half plane. Abouelsoud [8] generalized this result to both state and input constraints.

This paper is organized as follows. In Section 2 a state and control constrained controller is designed. Section 3 presents description of H. B. Robinson Nuclear power Plant model. In Section 4 simulation results and discussions are provided. Conclusion is given in Section 5.

2 Stabilization with State and Control Constraints

Given a continuous-time linear system

$$\dot{x} = Ax\,(t) + Bu\,(t)\,, \tag{1}$$

where $x \in R^n, u \in R^m, (A, B)$ is a controllable pair, and symmetric constraint state and control sets

$$S_x = \{x \in R^n : -w_x \le G_x x \le w_x\}\,, \tag{2}$$

$$S_u = \{u \in R^m : -w_u \le E_u u \le w_u\}\,. \tag{3}$$

By scaling we can make $w_x = \overline{1}$ and $w_u = \overline{1}$, where $\overline{1}$ is a column with elements unity,

$$S_x = \left\{x \in R^n : -\overline{1} \le G_x x \le \overline{1}\right\}, \tag{4}$$

$$S_u = \left\{u \in R^m : -\overline{1} \le E_u u \le \overline{1}\right\}, \tag{5}$$

$G_x \in R^{(s_1 \times n)}$, $E_u \in R^{(r_1 \times m)}$ are both full rank, we consider the problem of designing a linear state feedback controller

$$u\,(t) = Fx\,(t) \tag{6}$$

such that the closed loop system

$$\dot{x} = A_C x\,(t)\,, \tag{7}$$

where $A_C = A + BF$, is asymptotically stable and both the state and control constraints (4) and (5) are satisfied. We use the results of [8] to design F. First choose the closed loop poles according to the following criterion.

Lemma 2.1 [7] *A necessary and sufficient condition for*

$$S_x = \left\{x \in R^n : -\overline{1} \le G_x x \le \overline{1}\right\}$$

to be positively invariant for system (7) is that the eigenvalues $\lambda_i = \mu_i \pm j\sigma_i$ of matrix A_C satisfy

$$\mu_i \le -\left|\sigma_i\right|. \tag{8}$$

Proof See [7].

Let

$$G = \begin{bmatrix} G_x \\ 0 \end{bmatrix}, \quad E = \begin{bmatrix} 0 \\ E_u \end{bmatrix}.$$

Then the state and control constraints become

$$-\overline{1} \leq \begin{pmatrix} G \\ EF \end{pmatrix} x \leq \overline{1} \quad \text{or} \quad -\overline{1} \leq Gx + Eu \leq \overline{1}.$$

Assume that the invariant zeros of the system $\sum_1 : (A, b, G, E)$ are in the open left-half plane. (i.e. $\sum_1$ is minimum phase), then we can choose the closed loop poles as those invariant zeros; the remaining closed-loop poles are chosen to satisfy condition (8). Let λ_i be an invariant zero of $\sum_1$, then there exist a state direction v_i and a control direction w_i such that

$$P(\lambda_i) \begin{pmatrix} v_i \\ w_i \end{pmatrix} = \begin{pmatrix} \lambda_i I - A & -B \\ G & E \end{pmatrix} \begin{pmatrix} v_i \\ w_i \end{pmatrix} = 0. \tag{9}$$

for $i = 1, ..., n - s$, where $s = rank \begin{pmatrix} G_x B \\ E_u \end{pmatrix}$, $P(\lambda_i)$ is the system matrix. Hence the feedback matrix satisfies

$$F v_i = w_i \quad \text{or} \quad FV_1 = W_1, \tag{10}$$

where $V_1 = (v_1, ..., v_{n-s})$, $W_1 = (w_1, .., w_{n-s})$. The remaining closed-loop poles are chosen to satisfy conditions (8). Thus there exist closed-loop eigenvectors V_2 satisfying

$$GV_2 + EW_2 = I_{S \times S}, \tag{11}$$

$$V_2 \Lambda_2 = AV_2 + BW_2, \tag{12}$$

where

$$\Lambda_2 = blockdiag \begin{pmatrix} \mu_i & -\sigma_i \\ \sigma_i & \mu_i \end{pmatrix}.$$

For simple complex poles or real poles of the feedback matrix A_C, let

$$FV_2 = W_2. \tag{13}$$

Hence

$$F = \begin{pmatrix} W_1 & W_2 \end{pmatrix} \begin{pmatrix} V_1 & V_2 \end{pmatrix}^{-1}. \tag{14}$$

The feedback matrix F (14) ensures that the closed loop system is asymptotically stable and the state and control constraints are satisfied [8]. The control is now

$$u = Fx. \tag{15}$$

3 Robinson Nuclear Power Model

A linear differential equations of pressurized water reactor model that includes the reactor core, pressurizer, primary system piping, and a U-tube recirculation-type steam generator.

3.1 Core point kinetics equations

The point kinetics equations with six groups of delayed neutrons and reactivity feedbacks due to changes in fuel temperature, coolant temperature, and primary coolant system pressure. For model with one fuel node and two coolant nodes [3]

$$\frac{d\delta P}{dt} = -400\delta P + 0.0125\delta C_1 + 0.035\delta C_2 + 0.111\delta C_3 + 0.301\delta C_4 + 1.140\delta C_5 \\ +3.01\delta C_6 - 1781\delta T_f - 13700\delta T_{C1} - 13700\delta T_{C2} + 411\delta P_P + 10^6\delta\rho_{Rod}, \tag{16}$$

$$\frac{d\delta C_1}{dt} = 13.125\delta P - 0.0125\delta C_1, \tag{17}$$

$$\frac{d\delta C_2}{dt} = 87.5\delta P - 0.0305\delta C_2, \tag{18}$$

$$\frac{d\delta C_3}{dt} = 78.125\delta P - 0.111\delta C_3, \tag{19}$$

$$\frac{d\delta C_4}{dt} = 158.125\delta P - 0.301\delta C_4, \tag{20}$$

$$\frac{d\delta C_5}{dt} = 46.25\delta P - 1.140\delta C_5, \tag{21}$$

$$\frac{d\delta C_6}{dt} = 16.875\delta P - 3.01\delta C_6, \tag{22}$$

$$\frac{d\delta T_f}{dt} = 0.0756\delta P - 0.16466\delta T_f + 0.16466\delta T_{C1}, \tag{23}$$

$$\frac{d\delta T_{C1}}{dt} = 0.05707\delta T_f + 2.3832\delta T_{LP} - 2.4403\delta T_{C1}, \tag{24}$$

$$\frac{d\delta T_{C2}}{dt} = 0.05707\delta T_f - 2.3832\delta T_{C2} + 2.3262\delta T_{C1}. \tag{25}$$

3.2 Pressurizer equations

The pressurizer model is based on mass, energy, and volume balances with the assumption that saturation conditions always apply for steam-water mixture in the pressurizer,

$$\frac{d\delta P_P}{dt} = 0.0207\delta T_f - 0.0207\delta T_{C1} + 0.0103\delta T_{C2} + 0.240\delta T_{UP} - 0.130\delta T_{IP} \\ -0.509\delta T_P + 0.634\delta T_m - 0.116\delta T_{OP} + 0.121\delta T_{LP} - 0.279\delta T_{HL} \\ +0.0235\delta T_{CL} - 0.0062\delta Q. \tag{26}$$

3.3 Steam generator equations

The steam generator model with three regions: primary fluid, tupe metal, and secondary fluid,

$$\frac{d\delta T_P}{dt} = 0.2238\delta T_{IP} - 0.76642\delta T_P - 0.53819\delta T_m, \tag{27}$$

$$\frac{d\delta T_m}{dt} = 3.07017\delta T_P - 5.3657\delta T_m - 0.33272\delta P_s, \tag{28}$$

$$\frac{d\delta P_s}{dt} = 1.349\delta T_m - 0.2034\delta P_s - 0.0384\delta W_{FW}. \tag{29}$$

3.4 Piping equations

All piping sections are modeled as well-mixed volumes,

$$\frac{d\delta T_{UP}}{dt} = 0.33645\delta T_{C2} - 0.33645\delta T_{UP}, \tag{30}$$

$$\frac{d\delta T_{HL}}{dt} = 2.5\delta T_{UP} - 2.5\delta T_{HL} - 0.0016\delta W_P, \tag{31}$$

$$\frac{d\delta T_{IP}}{dt} = 1.45\delta T_{HL} - 1.45\delta T_{IP}, \tag{32}$$

$$\frac{d\delta T_{OP}}{dt} = 1.45\delta T_P - 1.45\delta T_{OP}, \tag{33}$$

$$\frac{d\delta T_{CL}}{dt} = 1.48\delta T_{OP} - 1.48\delta T_{CL}, \tag{34}$$

$$\frac{d\delta T_{LP}}{dt} = 0.516\delta T_{CL} - 0.516\delta T_{LP}, \tag{35}$$

where
δP: deviation in reactor power from its intial steady -state value,
δC_i: deviation of normalized precursor concentrations,
δT_f:deviation of fuel temperature in the fuel node,
δT_{C1}: deviation of coolant temperature in the first coolant node,
δT_{C2}: deviation of coolant temperature in the second coolant node,
δP_P: deviation of primary system pressure,
δT_P: deviation of temperature of primary coolant node in the steam generator,
δT_m: deviation of the steam generator tube metal temperature,
δP_s: deviation of steam pressure from its initial steady-state value,
δT_{UP}: deviation of the reactor upper plenum temperature,
δT_{LP}: deviation of the reactor lower plenum temperature,
δT_{HL}: deviation of hot leg temperature,
δT_{IP}: deviation of temperature of primary coolant in the steam generator or inlet plenum,
δT_{OP} : deviation of temperature of primary coolant in the steam generator or outlet plenum,
δT_{CL}: deviation of cold leg temperature,
$\delta \rho_{Rod}$: reactivity due to control rod movement,
δQ : rate of heat addition to the pressurizer fluid with electric heater,
δW_{FW}: deviation of feedwater flow rate in steam generator,
δW_P : deviation of primary water flow rate to the steam generator.

Eqs (16)–(35) describing the H. B. Robinson nuclear power system formed in state space model as follow:

$$\dot{x} = Ax + Bu, \tag{36}$$

where

$$x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T,$$

$$x_1 = \begin{bmatrix} \delta P & \delta C_1 & \delta C_2 & \delta C_3 & \delta C_4 & \delta C_5 & \delta C_6 & \delta T_f & \delta T_{C1} & \delta T_{C2} \end{bmatrix},$$

$$x_2 = \left[\begin{array}{cccccccccc} \delta P_P & \delta T_P & \delta T_m & \delta P_s & \delta T_{UP} & \delta T_{HL} & \delta T_{IP} & \delta T_{OP} & \delta T_{CL} & \delta T_{LP} \end{array} \right],$$

$$u = \left[\begin{array}{cccc} \delta \rho_{Rod} & \delta W_{FW} & W_P & \delta Q \end{array} \right]^T,$$

$$B = \left[\begin{array}{cccccccccccccccccccc}
10^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.03843 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0016 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -0.0062 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \right]^T,$$

and

$$A = \left[\begin{array}{cc} A_1 & A_2 \end{array} \right],$$

$$A_1 = \left[\begin{array}{cccccccccc}
-400 & 0.0125 & 0.0305 & 0.111 & 0.301 & 1.14 & 3.01 & -1781 & -13700 & -13700 \\
13.125 & -0.0125 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
87.5 & 0 & -0.0305 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
78.125 & 0 & 0 & -0.111 & 0 & 0 & 0 & 0 & 0 & 0 \\
158.125 & 0 & 0 & 0 & -0.301 & 0 & 0 & 0 & 0 & 0 \\
46.25 & 0 & 0 & 0 & 0 & -1.14 & 0 & 0 & 0 & 0 \\
16.875 & 0 & 0 & 0 & 0 & 0 & -3.01 & 0 & 0 & 0 \\
0.07 & 0 & 0 & 0 & 0 & 0 & 0 & -0.16466 & 0.16466 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05707 & -2.4403 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.05707 & 2.3262 & -2.3832 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0207 & -0.0207 & 0.0103 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.33645 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{array} \right],$$

$$A_2 = \left[\begin{array}{cccccccccc}
411 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2.3832 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.634 & -0.509 & 0 & 0.240 & -0.279 & -0.130 & -0.116 & 0.0235 & 0.121 \\
0 & -5.3657 & 3.07017 & 0.3372 & 0 & 0 & 0.2238 & 0 & 0 & 0 \\
0 & 0.53819 & -0.76442 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1.349 & 0 & -0.2034 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -0.33645 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2.5 & -2.5 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1.45 & -1.45 & 0 & 0 & 0 \\
0 & 1.45 & 0 & 0 & 0 & 0 & 0 & -1.45 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1.48 & -1.48 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.516 & -0.516
\end{array} \right].$$

We can apply the technique in (14) for designing a linear state feedback controller with state and control constraints to the system (36). The state constraints are on neutron flux level (δP), steam pressure in steam generator (δP_s) and hot leg temperature (δT_{HL}). Thus

$$-400 \le \delta P \le 400, \quad -10.07 \le \delta P_s \le 10.07, \quad -347.24 \le \delta T_{HL} \le 347.24.$$

By scaling we can make

$$-1 \le 0.0025 \delta P \le 1, \quad -1 \le 0.099 \delta P_s \le 1, \quad -1 \le 0.0021 \delta T_{HL} \le 1.$$

The control constraint depends on reactivity $(\delta\rho_{Rod})$ and electric heater to pressurizer (δQ),

$$-1 \leq 200\delta\rho_{Rod} + 0.006\delta Q \leq 1.$$

Thus,

$$G_x = \left[\begin{array}{cccccccccccccccccccc} 0.0025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.099 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0021 & 0 & 0 & 0 & 0 \end{array}\right],$$

$$G = \left[\begin{array}{cccccccccccccccccccc} 0.0025 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.099 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0021 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array}\right],$$

and

$$E_u = \left[\begin{array}{cccc} 200 & 0 & 0 & 0.006 \end{array}\right], \quad \text{rank}\left(\begin{array}{c} G_x B \\ E_u \end{array}\right) = 4.$$

Thus,

$$E = \left[\begin{array}{cccc} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 200 & 0 & 0 & 0.006 \end{array}\right].$$

The transfer function of system (36) has 16 zeros at -0.3365, -0.0849, -2.4362, -2.3832, -1.48, -1.45, -0.516, -0.1688, -5.701, -0.4315, -0.0305, -0.111, -0.301, -1.14, -3.01, -1.45, which are in left hand poles thus the transfer function is minimum phase.

The system matrix $P(\lambda) = \left(\begin{array}{cc} \lambda I - A & -B \\ G & E \end{array}\right)$ has 16 state direction v_i and 16 control direction $w_i, i = 1, ..., 16$.

Let $V_1 = \left[\begin{array}{cccc} v_1 & v_2 & \cdots & v_{16} \end{array}\right] = \left[\begin{array}{cc} v_{11} & v_{12} \end{array}\right]$, where

$$v_{11} = \left[\begin{array}{cccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3e-016 & -5e-012 & -2e-013 & -6e-011 & 3.6e-12 & -0.005 \\ 0 & 0 & -4e-010 & -4e-014 & -0.0006 & -0.0006 & 0.0003 & -3.6e-015 \\ 0 & 0 & 0.0022 & -0.0021 & -0.0015 & 0.0015 & 0.0003 & -0.0003 \\ 0.0006 & -0.0730 & -0.0025 & 0.0026 & 0.0041 & -0.0042 & -0.0039 & 0.0381 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -0.00006 & 0.00001 & -0.0003 & 0.0003 & 0.0004 & -0.0004 & -0.0006 & -0.0005 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -3e-015 & -6e-011 & 3e-019 & 4e-015 & 3e-012 & -6e-018 \\ 0 & 0 & -6e-011 & -8e-014 & 0.0004 & -0.0004 & 2e-011 & -2e-017 \\ 0 & 0 & 2e-013 & -3e-012 & -0.0002 & 0.0002 & 0.0002 & 6e-019 \end{array}\right],$$

$$v_{12} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ -8e-011 & 0.0001 & 0 & 0 & 0 & 0 & 0 & -0.0001 \\ -9e-015 & 0.0001 & 0 & 0 & 0 & 0 & 0 & -0.0006 \\ -3e-012 & 0.0002 & 0 & 0 & 0 & 0 & 0 & -0.0015 \\ -0.0035 & -0.0024 & 0.0001 & -0.0009 & -0.0003 & -0.0002 & -0.0002 & 0.0042 \\ -0.0285 & 6e-018 & 0 & 0 & 0 & 0 & 0 & 7e-018 \\ -0.0031 & 0.00002 & 0 & 0 & 0 & 0 & 0 & 6e-018 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -9e-018 & -0.0006 & 7e-019 & -6e-020 & 7e-018 & -6e-018 & -4e-012 & 0.0004 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -5e-011 \\ 0.0011 & 6e-017 & 0 & 0 & 0 & 0 & 0 & 6e-016 \\ -0.0004 & 8e-019 & 0 & 0 & 0 & 0 & 0 & 0.0004 \\ 2e-013 & 0.0001 & 0 & 0 & 0 & 0 & 0 & -0.0002 \end{bmatrix},$$

$$W_1 = \begin{bmatrix} w_1 & w_2 & \cdots & w_{16} \end{bmatrix} = \begin{bmatrix} w_{11} & w_{12} \end{bmatrix},$$

$$w_{11} = \begin{bmatrix} -6e-018 & 2e-016 & 1e-013 & -6e-015 & -2e-012 & 5e-014 & 6e-019 & 7e-018 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -8e-017 & 0.5418 & -0.5426 & -0.6841 & 0.6923 & 1 & 0.8253 \\ 0.0084 & -1 & -1 & 1 & 1 & -1 & -0.3402 & 1 \end{bmatrix},$$

$$w_{12} = \begin{bmatrix} 6e-018 & 6e-018 & -2e-016 & 1e-019 & 5e-012 & 4e-015 & 2e-011 & -2e-020 \\ 1 & 0.0002 & 0 & 0 & 0 & 0 & 0 & 6e-018 \\ 0.0019 & 1 & -4e-013 & -6e-017 & -5e-013 & 0 & -3e-019 & -0.6923 \\ -0.0438 & -0.1894 & 0.0006 & -0.0158 & -0.0140 & -0.0411 & -0.1032 & 1 \end{bmatrix}.$$

The corresponding state and control direction are 4 state direction v_i and 4 control direction $w_i, i = 17, ..., 20$.

$$V_2 = \begin{bmatrix} v_{17} & v_{18} & v_{19} & v_{20} \end{bmatrix} = \begin{bmatrix} 400 & -1.9e-016 & -6.9e-016 & 2.7e-015 \\ -769.6 & 3.6e-015 & -2.9e-015 & -1e-014 \\ -5144.4 & 6e-014 & -2.5e-014 & -7e-014 \\ -4648.2 & 2.2e-014 & -1e-013 & -6.1e-014 \\ -9681.6 & 1.2e-013 & -2.1e-013 & -1.3e-013 \\ -3249 & 4.2e-014 & -5.1e-014 & -5.1e-014 \\ -1765.2 & 2.1e-014 & -1.4e-014 & -8.2e-014 \\ -4.5327 & -7.9e-004 & 11e-004 & -6.1e-017 \\ 0.0589 & -0.0262 & 0.0054 & 3e-018 \\ 0.0273 & 0.0191 & -0.0023 & -3.3e-018 \\ -4.9772 & -2.9407 & 15.6583 & 0.2972 \\ -8.7e-021 & 23.5195 & -4.2434 & -2.9e-020 \\ 4.6e-021 & -2.6368 & 3.6039 & -1.7e-020 \\ -6.5e-022 & 10.10 & 2.6e-017 & 9.1e-021 \\ -0.0014 & -0.0012 & 9.9e-005 & 3.3e-019 \\ 7.1e-020 & -3.1e-020 & 476.1905 & -4.6e-019 \\ -8.8e-020 & -2.3e-021 & -105.8493 & 3.2e-019 \\ -1.5e-021 & 0.9287 & -0.8011 & 1.3e-020 \\ 2.5e-022 & -0.3363 & 0.1826 & -9.1e-021 \\ 6.6e-024 & 0.0344 & -0.0126 & 1.5e-021 \end{bmatrix}$$

and

$$W_2 = \begin{bmatrix} w_{17} & w_{18} & w_{19} & w_{20} \end{bmatrix} = \begin{bmatrix} 0.1650 & 0.0011 & -0.0064 & -1.22e-004 \\ -4.3e-019 & 2.2e+003 & -148.956 & -2.8e-019 \\ 2.2128 & 1.9233 & -1.6289e+006 & -2.2e-016 \\ -5501.1 & -37.0275 & 213.0615 & 170.7381 \end{bmatrix}.$$

The remaining closed loop poles are chosen as follows: -6.834 , -5.567 , -7.9732 , -0.15. The state and control feedback controller

$$F = \begin{pmatrix} W_1 & W_2 \end{pmatrix} \begin{pmatrix} V_1 & V_2 \end{pmatrix}^{-1}, \tag{37}$$

$$F = \left[\begin{array}{ccc} F_1 & F_2 & F_3 \end{array} \right],$$

$$F_1 = \left[\begin{array}{ccccccc} 3.8e-4 & -7.6e-6 & -3e-8 & -1e-7 & -3e-7 & -1.1e-6 & -3e-6 \\ 2e-4 & 1.01e-4 & 0 & 0 & 0 & 0 & 0 \\ 11.70 & 6.08 & 6.8e-18 & 5e-16 & 1.8e-16 & 7e-16 & 2.6e-15 \\ -6.5e+16 & -3.4e+16 & 0.001 & 0.0037 & 0.01 & 0.038 & 0.1003 \end{array} \right],$$

$$F_2 = \left[\begin{array}{ccccccc} 0.0018 & 0.0137 & 0.0137 & -4.1e-4 & -6e-16 & -2.6e-15 & 8.1e-17 \\ 0 & 0 & 0 & 0 & 35.10 & 0 & 139.56 \\ -2.9e-11 & 8e-10 & 1.5e-12 & 9.5e-14 & 8.1e-10 & 6e-9 & 7.1e-11 \\ -59.36 & -456.66 & -456.66 & 13.7 & 1.1e-11 & 5.1e-11 & 11.006 \end{array} \right],$$

$$F_3 = \left[\begin{array}{cccccc} 0 & 1.1e-7 & 5.9e-7 & 3.1e-15 & 4e-17 & 1.2e-16 \\ 0 & 4.4e-16 & 9e-16 & 0 & 0 & 0 \\ -1562.5 & -3420.8 & -0.38 & -4.5e-9 & 6e-10 & -1e-9 \\ 0 & -0.30 & -0.0039 & -6.9e-11 & 2.6e-13 & -7.1e-12 \end{array} \right].$$

4 Result and Discussions

This section presents the simulation and numerical results based on linear state feedback controller (37) applied to the system (36). The system is simulated for initial state variables values as follows

$$x = \left[\begin{array}{cccccccccccccccccccc} 400 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 10.07 & 0 & 347.24 & 0 & 0 & 0 & 0 \end{array} \right],$$

where the initial values satisfy the defined constraints of deviations of neutron flux, steam pressure and hot leg temperature.

The following figures represent the responses (deviations of the system state variables with time, where it is clear that all deviations decay with time and tend to zero, satisfying both performance criterion stability and zero steady state error (1 sec=1000 iterations).

Figure 1 shows deviation of neutron flux with time, we observe the curve has speed response (maximum overshoot is within the acceptable constraint $400 \leq x_1 \leq 400$).

Figure 2 shows deviation of generations from the first to sixth of the delayed neutron fractions within range [-150, 300]. It is clear that the increase in the maximum overshoot of responses as the increase of generations of the delayed neutron fractions, while the damping frequency and the settling time of all delayed neutron fractions are the same.

Figure 3 shows the deviation of fuel temperature, it is clear that from graph the maximum over shoot are very small and settling time (≤ 400 iterations).

Figure 4 shows the deviations of coolant temperature in first node and second node, also show the deviations of metal and primary temperatures in steam generator, we observe the curves have speed response, maximum overshoot is within the range [0,5].

Figure 5 shows the deviation of primary pressure, it is clear that from graph the maximum over shoot is within the interval [0,150], decays with time and tends to zero.

Figure 6 shows the deviation of steam pressure in steam generator, we observe the curve has speed response (maximum overshoot is within the acceptable constraint $10.07 \leq x_{14} \leq 10.07$ and settling time (≤ 800 iterations).

Figure 7 shows the deviations of the reactor upper, outlet plenum, cold leg and lower plenum temperatures in steam generator, we observe the figures have the same range, the maximum over shoot are small and settling time (≤ 14000 iterations).

Figure 8 shows the deviation of hot leg temperature, it is clear that the graph has speed response with (maximum overshoot is within the acceptable constraint $-347.24 \leq x_{16} \leq 347.24$, and very small settling time (≤ 600 iterations).

Figure 9 shows deviation of inlet plenum temperature, we observe the figure has maximum over shoot are small, also its decays with time and tends to zero.

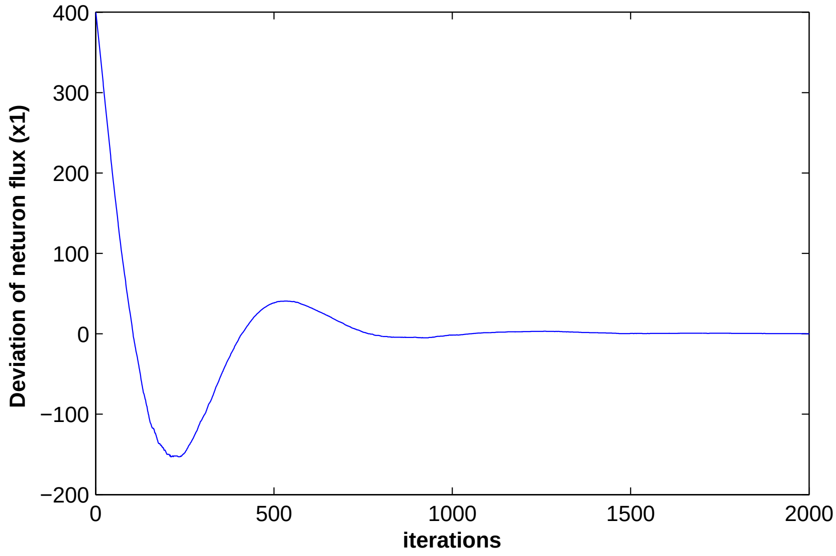

Figure 1: Deviation of neutron flux for simulations with the state feedback controller (37).

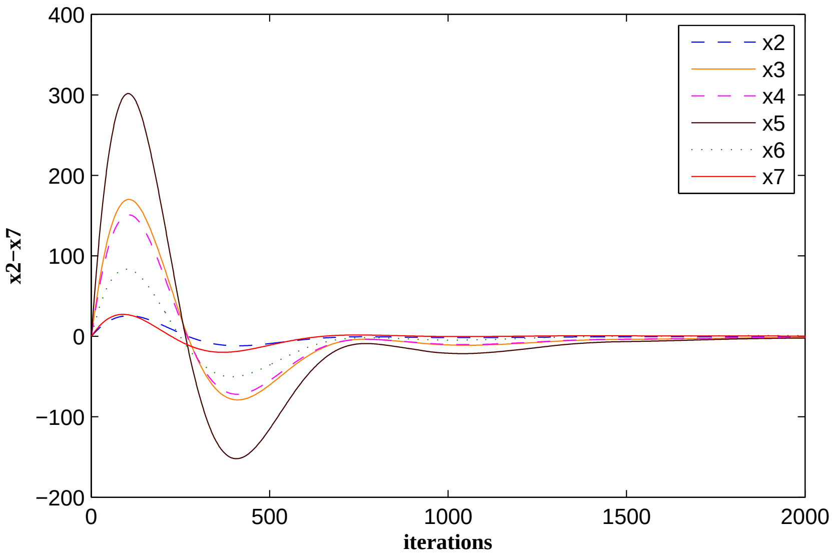

Figure 2: Deviation of normalized precursor concentrations for simulations with the state feedback controller (37).

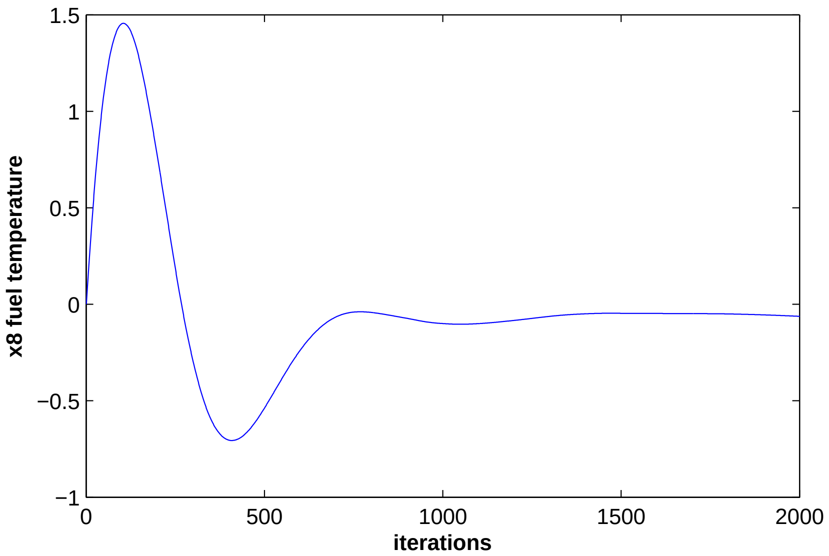

Figure 3: Deviation of fuel temperature for simulations with the state feedback controller (37).

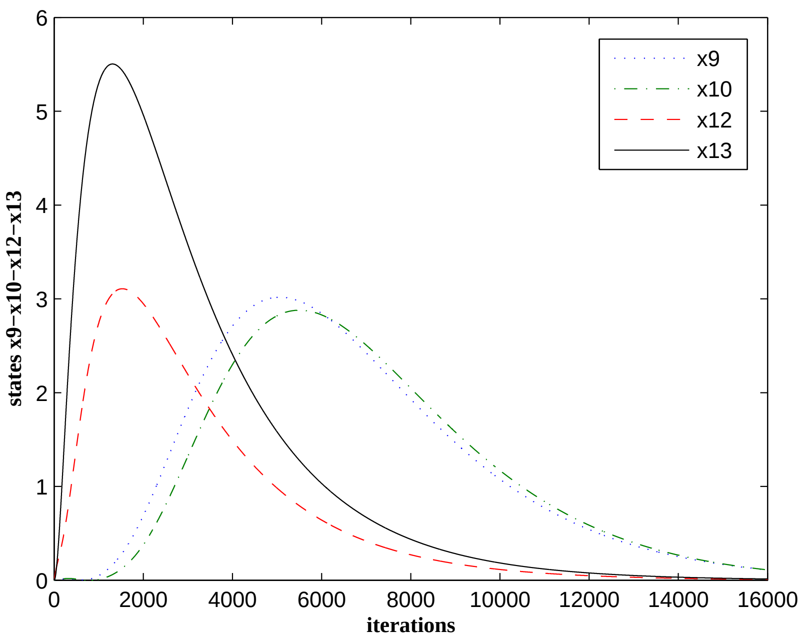

Figure 4: Deviations of coolant in first node ,second node , metal and primary steam generator temperatures for feedback controller (37).

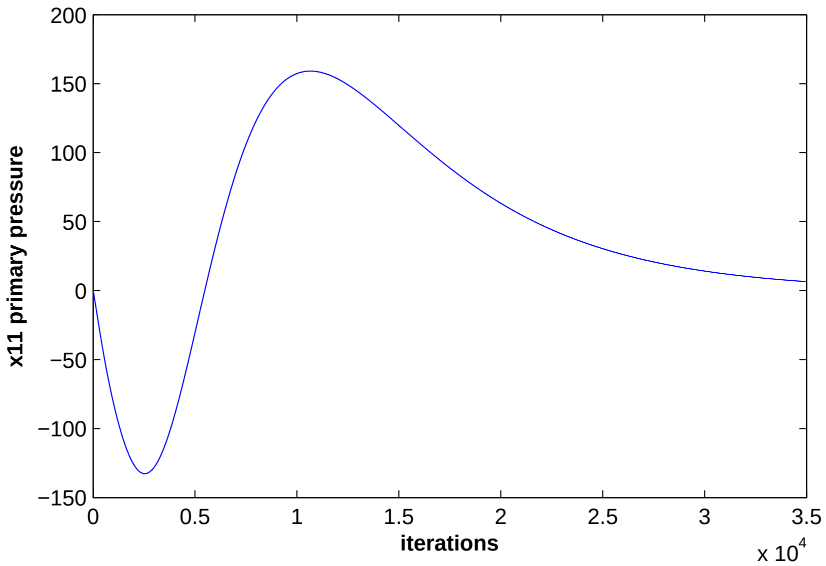

Figure 5: Deviation of primary pressure for simulations with the state feedback controller (37).

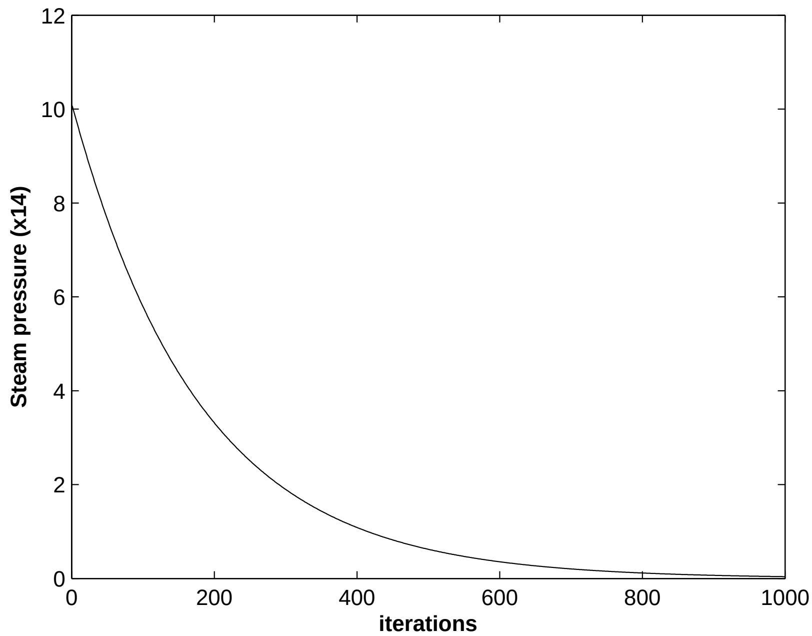

Figure 6: Deviation of steam pressure for simulations with the state feedback controller (37).

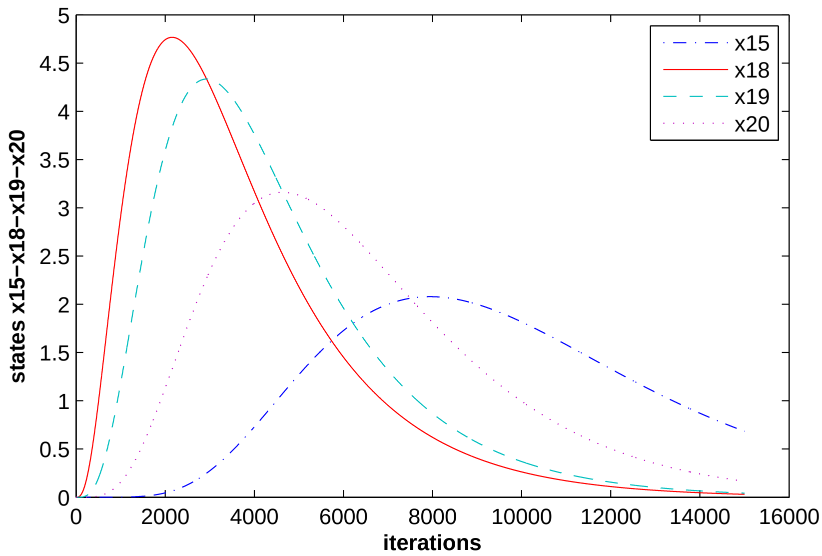

Figure 7: Deviation of the reactor upper, outlet plenum ,cold leg and lower plenum temperatures for controller (37).

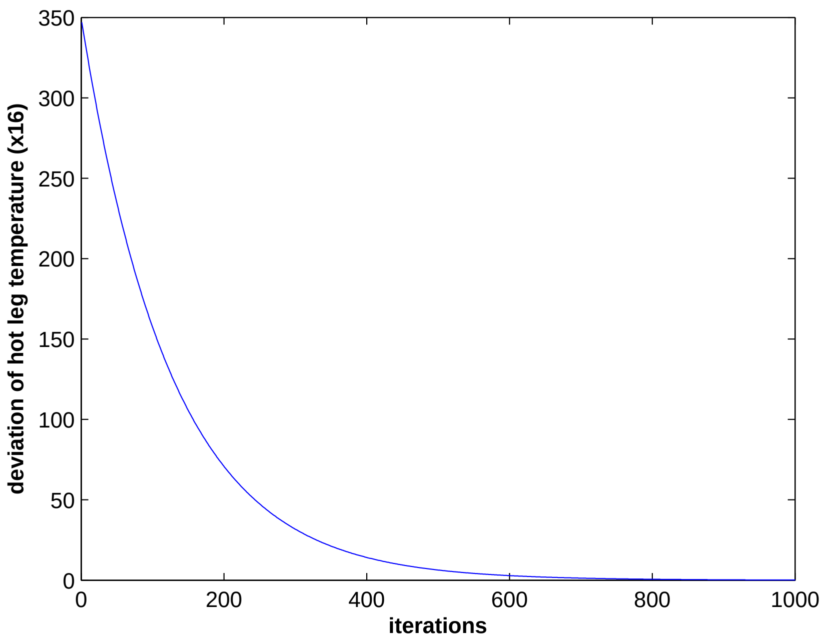

Figure 8: Deviation of hot leg temperature for simulations with the state feedback controller (37).

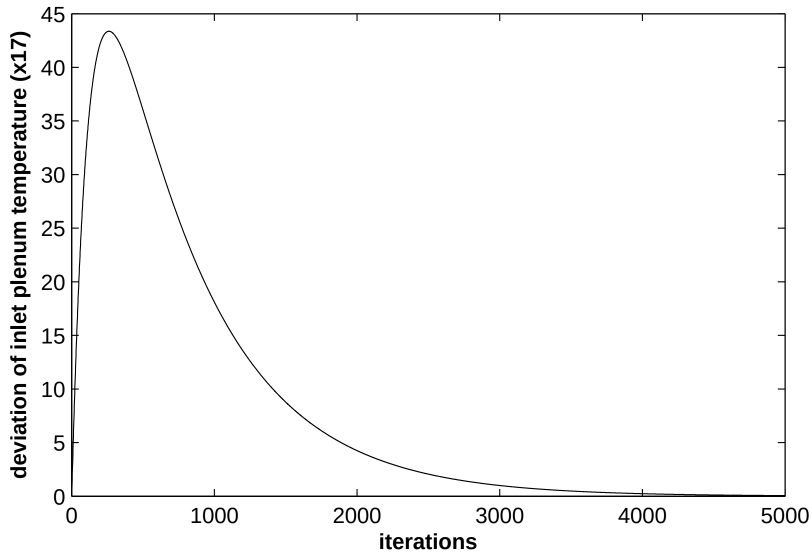

Figure 9: Deviation of inlet plenum temperature for simulations with controller (37).

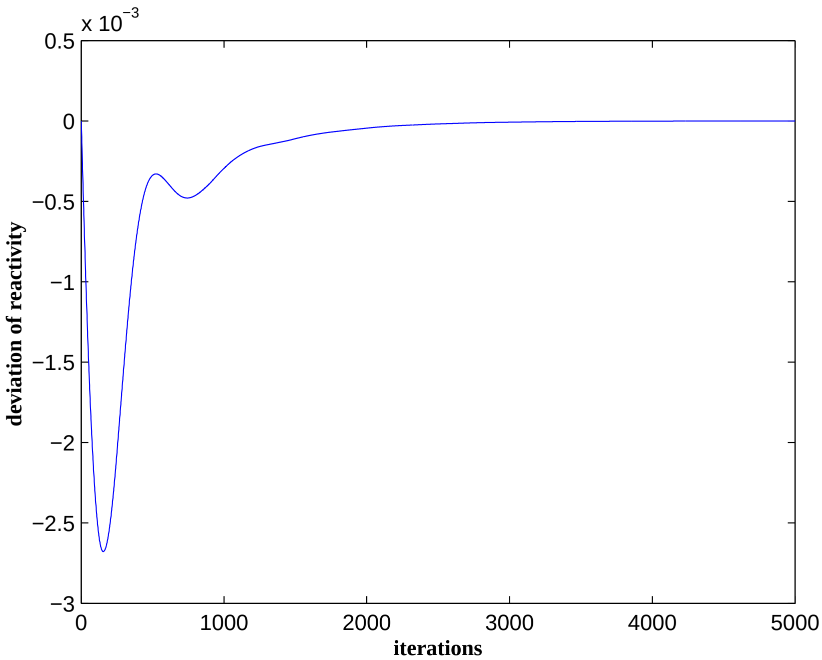

Figure 10: Deviation of reactivity for a simulation with the state feedback controller (37).

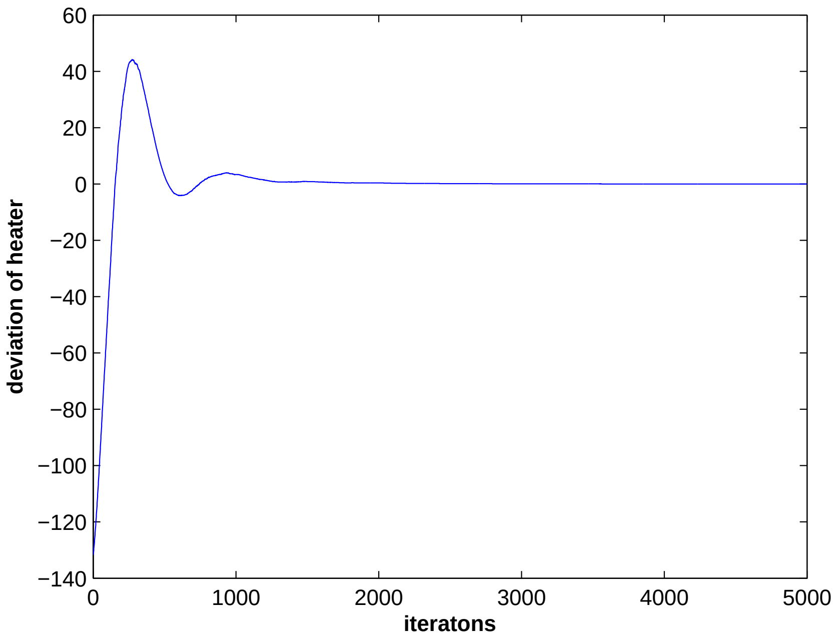

Figure 11: Deviation of electric heater input to the pressurizer with the state feedback controller (37).

Figures 10 and 11 show the deviations of reactivity, electric heater control inputs where the two curves satisfy the acceptable constraints, $-0.005 \leq \delta\rho_{Rod} \leq 0.005$, $-170 \leq \delta Q \leq 170$ which interprets the superior effect of the control technique used.

These figures demonstrate stability of the state feedback system, while the neutron flux level, steam pressure in steam generator, hot leg temperature, control input reactivity and control input electric heater to pressurizer constraints are satisfied.

The simulation results provide us with an important practical implication, that is the nuclear power plant has reached its desired steady state value in a very small time as the neutron flux of our theoretical system under study as shown in Figure 1 has reached the desired steady state value in about, which indicates a relative importance of our control algorithm for practical implementation of different systems.

5 Conclusions

A linear state feedback controller [9, 10] has been designed to globally asymptotically stabilize H.B. Robinson pressurized water reactor plant [2] subject to symmetrical neutron level flux, steam pressure in steam generator, hot leg temperature, control input reactivity and electric heater input constraints. Simulation results show the effectiveness of the proposed technique.

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