Existence and Uniqueness of Solutions to Quasilinear Integro-differential Equations by the Method of Lines

Jaydev Dabas

Department of Paper Technology, Indian Institute of Technology Roorkee, Saharanpur Campus, Saharanpur-247001, India.

Received: January 28, 2011; Revised: September 22, 2011

Abstract: In this work we consider a class of quasilinear integro-differential equations. We apply the method of lines to establish the wellposedness for a strong solution. The method of lines is a powerful tool for proving the existence and uniqueness of solutions to evolution equations. This method is oriented towards the numerical approximations.

Keywords: method of lines; integro-differential equation; semigroups; contractions; strong solution.

Mathematics Subject Classification (2000): 34K30, 34G20, 47H06.

1 Introduction

Let $X$ and $Y$ be two real reflexive Banach spaces such that $Y$ is densely and compactly embedded in $X$. In the present analysis we are concerned with the following quasilinear integro-differential equation

\[
\begin{aligned}
\frac{du}{dt}(t) + A(t, u(t))u(t) &= \int_0^t k(t, s)A(s, u(s))u(s)ds + f(t, u), \quad 0 < t \leq T, \\
\quad u_0 &= \phi \in C([-T, 0], X),
\end{aligned}
\]

(1)

where $A(t, u)$ is a linear operator in $X$, depending on $t$ and $u$, defined on an open subset $W$ of $Y$. We denote by $J = [0, T]$, $k$ is a real valued function defined on $J \times J \to \mathbb{R}$ and $f$ is defined from $J \times C([-T, 0], X)$ into $Y$. Here $C([a, b], Z)$, for $-\infty \leq a \leq b < \infty$, is the...