



# Existence and Uniqueness of Solutions to Quasilinear Integro-differential Equations by the Method of Lines

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**Abstract:** In this work we consider a class of quasilinear integro-differential equations. We apply the method of lines to establish the wellposedness for a strong solution. The method of lines is a powerful tool for proving the existence and uniqueness of solutions to evolution equations. This method is oriented towards the numerical approximations.

**Keywords:** *method of lines; integro-differential equation; semigroups; contractions; strong solution.*

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## 1 Introduction

Let  $X$  and  $Y$  be two real reflexive Banach spaces such that  $Y$  is densely and compactly embedded in  $X$ . In the present analysis we are concerned with the following quasilinear integro-differential equation

$$\begin{cases} \frac{du}{dt}(t) + A(t, u(t))u(t) = \int_0^t k(t, s)A(s, u(s))u(s)ds + f(t, u_t), & 0 < t \leq T, \\ u_0 = \phi \in C([-T, 0], X), \end{cases} \quad (1)$$

where  $A(t, u)$  is a linear operator in  $X$ , depending on  $t$  and  $u$ , defined on an open subset  $W$  of  $Y$ . We denote by  $J = [0, T]$ ,  $k$  is a real valued function defined on  $J \times J \rightarrow \mathbb{R}$  and  $f$  is defined from  $J \times C([-T, 0], X)$  into  $Y$ . Here  $C([a, b], Z)$ , for  $-\infty \leq a \leq b < \infty$ , is the

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