



# Cone Inequalities and Stability of Dynamical Systems

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Received: April 25, 2010; Revised: July 21, 2011

**Abstract:** The paper is devoted to working out new methods for stability analysis of equilibrium states of nonlinear dynamic systems in a partially ordered space. The concerned classes of differential systems are described by operator inequalities and inclusions using the notion of derivative with respect to a cone of nonlinear operator. Sufficient stability conditions of equilibrium states are formulated for sets of nonlinear and pseudolinear systems with the interval and polyhedral types operator coefficients. More general result is presented in the form of comparison principle for a finite set of differential systems.

**Keywords:** *dynamic system; pseudolinear system; monotone system; positive system; Lyapunov stability; cone inequality; partially ordered space.*

**Mathematics Subject Classification (2000):** Primary: 34D20, 47H07;  
Secondary: 34C12, 47A50.

## 1 Introduction

Stability analysis for dynamic systems with parameter or functional uncertainties is one of the fundamental issues in system and control theory. The applied researches employ continuous and discrete models of dynamic objects whose states possess certain properties with respect to a cone in the phase space (positivity, monotonicity, cooperativity, etc.). For example, these properties can be determined very often by using a cone of nonnegative vectors, a cone of symmetric nonnegatively definite matrices, an ellipsoidal cone, etc. Many important advances have been achieved on the basis of the operator theory in partially ordered spaces (see, e.g., [1–8]). In addition, classes of positive and monotone systems arise in stability theory as systems of comparison [7, 9–11].

We study generalized classes of positive and monotone dynamic systems with respect to a cone and give characterization for such systems by means of operator inequalities and

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