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Passive Delayed Static Output Feedback Control for a Class of T-S Fuzzy Systems

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Abstract: This paper investigates the problem of passive delayed static output feedback control for a class of fuzzy systems. The system is described by a state-space Takagi–Sugeno (T-S) fuzzy model with additive delays and interval parameter uncertainties. The aim is to design a fuzzy delayed static output feedback controller which ensures the closed-loop system is passive for all admissible uncertainties. In terms of linear matrix inequalities, a delay-dependent condition for the solvability of the above passive control problem is presented. A simulation example is provided to illustrate the effectiveness of the proposed design approach.

Keywords: *passive control; static output feedback; additive delays; T-S fuzzy models; interval parameter uncertainties.*

Mathematics Subject Classification (2000): 93C42, 93D09, 93D15.

1 Introduction

It is known that Takagi-Sugeno (T-S) fuzzy model, which is described by IF-THEN rules, provides an effective way to represent complex nonlinear systems in terms of fuzzy sets linear sub-systems [1, 13]. Time delays are commonly encountered in various engineering systems. Considerable attention has been paid to the stability analysis and synthesis for T-S fuzzy systems with time delays [12, 16], these results can be classified into two categories, namely, delay independent and delay dependent results. In most of these works, the state vector has a single delay. In this paper, we consider a class of T-S

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fuzzy systems with additive time-varying delays with totally different properties. Such a system model is suitable in the analysis of networked control systems [11].

Recently, passive control has attracted lots of attention among control community [9, 14]. For example, some results on passive control for T-S fuzzy systems were obtained for discrete- and continuous-time systems in [1] and [7], respectively. However, many papers deal with state feedback controllers. In practical applications, state variables may not be measured for many nonlinear systems. So it is meaningful to control a system via output feedback controllers; static output feedback control strategy is simple in controller structures, compared with dynamic output feedback control strategy. The problem of static output feedback controller design for discrete-time T-S fuzzy systems was considered in [2, 3], while for continuous-time T-S fuzzy systems, the static output feedback controller design problem was investigated in [4]. It is worth mentioning that, the delayed feedback control approach has attracted much attention over the past several years [8]. One can get rid of the need for explicitly determining any information about the underlying dynamics other than the period of the desired orbit, by using time delay in the feedback loop [6, 10]. However, to the best of our knowledge, the problem of passive delayed static output feedback control for continuous-time T-S fuzzy systems with interval parameter uncertainties and additive delays has not been solved.

In this paper, we consider the passive delayed static output feedback control problem for a class of fuzzy systems with uncertain parameters and delays. The purpose is to design a full-order fuzzy delayed static output feedback controller such that the resulting closed-loop system is passive irrespective of the parameter uncertainties. A sufficient condition for the solvability of this problem is proposed and an explicit expression of a desired static output feedback controller is also given.

Notation: Throughout this paper, for real symmetric matrices X and Y, the notation $X \geq Y$ (respectively, $X > Y$) means that the matrix $X - Y$ is positive semidefinite (respectively, positive definite). I and 0 denote the identity and the zero matrix with appropriate dimensions. $*$ is used as an ellipsis for terms induced by symmetry. Matrices, if not explicitly stated, are assumed to have compatible dimensions. $Sym(X)$ denotes the expression $X + X^T$.

2 Main Results

The Takagi-Sugeno (T-S) fuzzy dynamic model is described by fuzzy IF-THEN rules, which locally represent linear input-output relations of nonlinear systems. A continuous-time T-S fuzzy model with additive delays and interval parameter uncertainties can be described by

Plant Rule i: IF $s_1(t)$ is μ_{i1} and ... and $s_p(t)$ is μ_{ip}, THEN

$$\begin{aligned}
\dot{x}(t) &= A_i x(t) + A_{di} x(t - \tau_1(t) - \tau_2(t)) + B_i u(t) + D_{1i} w(t), && (1)\\
y(t) &= C_i x(t), && (2)\\
z(t) &= E_i x(t) + D_{2i} w(t) + E_{1i} u(t), && (3)\\
x(t) &= \phi(t) \quad \forall t \in [-\bar{\tau}_{12}, 0], \quad i = 1, 2, \ldots, r, && (4)
\end{aligned}$$

where μ_{ij} is the fuzzy set and r is the number of IF-THEN rules; $s_1(t)$, ..., $s_p(t)$ are the premise variables. Throughout this paper, it is assumed that the premise variables do not depend on control variables; $x(t) \in \mathbb{R}^n$ is the state; $u(t) \in \mathbb{R}^m$ is the control input; $y(t) \in \mathbb{R}^s$ is the measured output; $z(t) \in \mathbb{R}^q$ is the controlled output; $w(t) \in \mathbb{R}^p$ is the noise signal; $\tau(t)$ is the time delays in state either constant or time varying satisfying $0 \le \tau_i(t) \le \bar{\tau}_i,\ 0 \le \dot{\tau}_i(t) \le d_i,\ i = 1, 2$, where $\bar{\tau}_i$ and d_i are constants. For simplicity, set

$$\bar{\tau}_{12} = \bar{\tau}_1 + \bar{\tau}_2,\ d_{12} = d_1 + d_2.$$

For all $1 \le p, q \le n,\ 1 \le k \le n_B$ with $\underline{A}_i = [\underline{a}_i^{pq}],\ \overline{A}_i = [\overline{a}_i^{pq}],\ \underline{A}_{di} = [\underline{a}_{di}^{pq}],\ \overline{A}_{di} = [\overline{a}_{di}^{pq}],\ \underline{B}_i = [\underline{b}_i^{pk}],\ \overline{B}_i = [\overline{b}_i^{pk}]$, we define the following interval uncertain matrix sets:

$$\begin{aligned}
\mathcal{A}_i &= \{[a_i^{pq}]_{n\times n} : \underline{a}_i^{pq} \le a_i^{pq} \le \overline{a}_i^{pq}, 1 \le p, q \le n\},\\
\mathcal{A}_{di} &= \{[a_{di}^{pq}]_{n\times n} : \underline{a}_{di}^{pq} \le a_{di}^{pq} \le \overline{a}_{di}^{pq}, 1 \le p, q \le n\},\\
\mathcal{B}_i &= \{[b_i^{pk}]_{n\times n} : \underline{b}_i^{pk} \le b_i^{pk} \le \overline{b}_i^{pk}, 1 \le p \le n, 1 \le k \le n_B\},
\end{aligned}$$

and let $A_i \in \mathcal{A}_i,\ A_{di} \in \mathcal{A}_{di},\ B_i \in \mathcal{B}_i$, for $i = 1, 2, \ldots, r$.

Now, let

$$\begin{aligned}
A_{0i} &= \frac{1}{2}(\underline{A}_i + \overline{A}_i),\ \Delta A_i = \frac{1}{2}(\overline{A}_i - \underline{A}_i),\ A_{d0i} = \frac{1}{2}(\underline{A}_{di} + \overline{A}_{di}),\\
\Delta A_{di} &= \frac{1}{2}(\overline{A}_{di} - \underline{A}_{di}),\ B_{0i} = \frac{1}{2}(\underline{B}_i + \overline{B}_i),\ \Delta B_i = \frac{1}{2}(\overline{B}_i - \underline{B}_i).
\end{aligned}$$

Then $A_i,\ A_{di}$ and B_i in (1) can be rewritten as

$$\begin{aligned}
A_i &= A_{0i} + \sum_{p,q=1}^{n} e_p \left|g_{a_i}^{pq}\right| e_q^T,\ A_{di} = A_{d0i} + \sum_{p,q=1}^{n} e_p \left|g_{a_{di}}^{pq}\right| e_q^T,\\
B_i &= B_{0i} + \sum_{p=1}^{n}\sum_{k=1}^{n_B} e_p \left|g_{b_i}^{pk}\right| e_k^T,
\end{aligned}$$

where $\sum_{p,q=1}^{n} e_p \left|g_{a_i}^{pq}\right| e_q^T,\ \sum_{p,q=1}^{n} e_p \left|g_{a_{di}}^{pq}\right| e_q^T$, and $\sum_{p=1}^{n}\sum_{k=1}^{n_B} e_p \left|g_{b_i}^{pk}\right| e_k^T$ denote the interval parameter uncertainties; $e_p, e_q \in R^n$ and $e_k \in R^{n_B}$ are the column vectors with pth, qth, kth element to be 1 and others to be 0; $g_{a_i}^{pq},\ g_{a_{di}}^{pq}$, and $g_{b_i}^{pk}$ are variant parameters satisfying $\left|g_{a_i}^{pq}\right| \le \Delta a_i^{pq},\ \left|g_{a_{di}}^{pq}\right| \le \Delta a_{di}^{pq}$, and $\left|g_{b_i}^{pk}\right| \le \Delta b_i^{pk}$, respectively.

Then the final output of the fuzzy system is inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(s(t))[A_i x(t) + A_{di} x(t - \tau_1(t) - \tau_2(t)) + B_i u(t) + D_{1i} w(t)], \tag{5}$$

$$y(t) = \sum_{i=1}^{r} h_i(s(t))[C_i x(t)], \tag{6}$$

$$z(t) = \sum_{i=1}^{r} h_i(s(t))[E_i x(t) + D_{3i} w(t) + E_{1i} u(t)], \tag{7}$$

where

$$\begin{aligned}
h_i(s(t)) &= \frac{\varpi_i(s(t))}{\sum_{i=1}^{r} \varpi_i(s(t))},\quad \varpi_i(s(t)) = \prod_{j=1}^{p} \mu_{ij}(s_j(t)),\\
s(t) &= [s_1(t) \quad s_2(t) \quad \cdots \quad s_p(t)],
\end{aligned}$$

in which $\mu_{ij}(s_j(t))$ is the grade of membership of $s_j(t)$ in μ_{ij}. Then, it can be seen that

$$\begin{aligned} h_i(s(t)) &\geq 0, \quad i = 1, \cdots, r, \\ \sum_{i=1}^{r} h_i(s(t)) &= 1, \quad \forall t. \end{aligned} \tag{8}$$

Now, by the parallel distributed compensation, we consider the following full-order fuzzy delayed static output feedback controller for the fuzzy system (5)–(7):

$$u(t) = \sum_{i=1}^{r} h_i(s(t))[K_i y(t-\tau)], \tag{9}$$

where K_i is matrix to be determined later and τ is a given scalar.

From (5)–(7) and (9), the closed-loop system can be obtained as

$$\begin{aligned} \dot{x}(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(s(t))h_j(s(t))h_l(s(t))[A_i x(t) + B_i K_j C_l x(t-\tau) \\ &\quad + A_{di} x(t - \tau_1(t) - \tau_2(t)) + D_{1i} w(t)], \qquad (10) \\ z(t) &= \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(s(t))h_j(s(t))[E_i x(t) + E_{1i} K_j C_l x(t-\tau) + D_{3i} w(t)]. \qquad (11) \end{aligned}$$

As a performance measure for T-S fuzzy system (10)–(11), the definition of passivity is as follows:

Definition 2.1 [5] The system (10)–(11) is called passive if there exists a scalar $\gamma \geq 0$ such that

$$2\int_0^t \omega(t)^T z(t) dt \geq -\gamma \int_0^t \omega(s)^T \omega(s) ds \tag{12}$$

for all $t \geq 0$ and for all solutions of (10)–(11) with $x_0 = 0$, where γ is some constant which depends on the initial condition of the system.

2.1 Passivity analysis

We first give the following results which will be used in the proof of our main results.

Lemma 2.1 [15] *Given matrices $\mathcal{X} = \mathcal{X}^T$, $\mathcal{D}$, $\mathcal{Z}$ and $\mathcal{R} = \mathcal{R}^T > 0$ of appropriate dimensions, we have*

$$\mathcal{X} + \mathcal{D}\mathcal{F}\mathcal{Z} + \mathcal{Z}^T\mathcal{F}^T\mathcal{D}^T < 0$$

for all $\mathcal{F}$ satisfying $\mathcal{F}^T\mathcal{F} \leq \mathcal{R}$ if and only if there exists a scalar $\epsilon > 0$ such that

$$\mathcal{X} + \epsilon\mathcal{D}\mathcal{D}^T + \epsilon^{-1}\mathcal{Z}^T\mathcal{R}\mathcal{Z} < 0.$$

Theorem 2.1 *Consider the closed-loop fuzzy system in (5)–(7) with interval parameter uncertainties and additive delays. Suppose that the controller gain matrices in (9) are known. Given positive scalars γ, d_1, d_{12}, $\bar{\tau}_1$, $\bar{\tau}_2$ and $\bar{\tau}_{12}$, if there exist matrices $P > 0$, $Q > 0$, $R_i > 0$, $Z_i > 0$, $M_i > 0$, S, and positive scalars ε_{1ijpq}, ε_{2ijpq}, ε_{3ijpq}, ε_{4ijpq}, ε_{5ijpk}, ε_{6ijpk}, for $p, q = 1, \ldots, n$ and $k = 1, \ldots, n_B$, such that the following linear matrix inequalities (LMIs) hold:*

$$\begin{aligned}
\Psi_{iil} &< 0,\ i,l=1,\ldots,r, &(13)\\
\Psi_{ijl}+\Psi_{jil} &< 0,\ 1\le i<j\le r, l=1,\ldots,r, &(14)\\
M_k &< Z_k,\ k=1,2,3, &(15)
\end{aligned}$$

where

$$\Psi_{ijl}=\begin{bmatrix} \Omega_{ijl} & \Omega_s & \Omega_s & \Delta_{a_i}^{pq}\Omega_e & \Delta_{a_{di}}^{pq}\Omega_e & \Delta_{b_i}^{pk}\Omega_e \\ * & -\Omega_{\varepsilon 1ij}-\Omega_{\varepsilon 2ij} & 0 & 0 & 0 & 0 \\ * & * & -\Omega_{\varepsilon 5ijB} & 0 & 0 & 0 \\ * & * & * & -\Omega_{\varepsilon 3ij} & 0 & 0 \\ * & * & * & * & -\Omega_{\varepsilon 4ij} & 0 \\ * & * & * & * & * & -\Omega_{\varepsilon 6ijB} \end{bmatrix},$$

$$\begin{aligned}
\Omega_{ijl} &= \Phi_{0ijl}+\sum_{p,q=1}^{n}\Big[(\varepsilon_{1ijpq}\Delta_{a_i}^{pq^2}+\varepsilon_{3ijpq})W_{eq1}^TW_{eq1}+(\varepsilon_{2ijpq}\Delta_{a_{di}}^{pq^2}+\varepsilon_{4ijpq})W_{eq2}^TW_{eq2}\Big]\\
&\quad+\sum_{p=1}^{n}\sum_{k=1}^{n_B}\Big(\varepsilon_{5ijpk}\Delta_{b_i}^{pk^2}+\varepsilon_{6ijpk}\Big)W_{ek1}^TW_{ek1},
\end{aligned}$$

$$\begin{aligned}
W_{eq1} &= \begin{bmatrix} e_q^T & 0_{1,(m+3)n}\end{bmatrix},\ W_{ek1}=\begin{bmatrix} 0_{1,n} & e_k^TK_jC_l & 0_{1,(m+2)n}\end{bmatrix},\\
W_{eq2} &= \begin{bmatrix} 0_{1,3n} & e_q^T & 0_{1,mn}\end{bmatrix},\Omega_s=\begin{bmatrix}\bar{S}e_1 & \cdots & \bar{S}e_n\end{bmatrix},\\
\Omega_e &= \begin{bmatrix}\bar{e}_1S^T & \cdots & \bar{e}_nS^T\end{bmatrix},\ \bar{e}_pS=\begin{bmatrix}0_{1,mn} & e_p^TS^T & 0_{1,3n}\end{bmatrix},\\
\bar{S}e_p &= \begin{bmatrix} Se_p \\ 0_{(m+3)n,1}\end{bmatrix},\ \Omega_{\varepsilon nij}=\begin{bmatrix}\varepsilon_{nij11} & 0 & 0\\ * & \ddots & 0\\ * & * & \varepsilon_{nijnn}\end{bmatrix},\\
\Omega_{\varepsilon mijB} &= \begin{bmatrix}\varepsilon_{mij11} & 0 & 0\\ * & \ddots & 0\\ * & * & \varepsilon_{mijnn_B}\end{bmatrix},\Phi_{0ijl}=\begin{bmatrix}\Sigma_{01ijl} & \Sigma_{02ijl} & \Sigma_4\\ * & \Sigma_{03i} & \Sigma_5\\ * & * & -\Sigma_6\end{bmatrix},\\
\Sigma_{01ijl} &= \begin{bmatrix} Q_{A0i} & SB_{0i}K_jC_l+L_{12}^T+L_{32}^T & L_{21}-L_{11}+L_{13}^T+L_{33}^T\\ * & -Q & L_{22}-L_{12}\\ * & * & (d_1-1)R_1+R_2+Sym\{L_{23}-L_{13}\}\end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\Sigma_{02ijl} &= \begin{bmatrix} SA_{d0i}+\hat{L}_{21} & SD_{1i}-E_i^T+L_{15}^T+L_{35}^T & P-S+A_{0i}^TS^T+L_{16}^T+L_{36}^T\\ -L_{22}-L_{32} & -C_l^TK_j^TE_{1i}^T & C_l^TK_j^TB_{0i}^TS^T\\ \hat{L}_{23} & L_{25}^T-L_{15}^T & L_{26}^T-L_{16}^T\end{bmatrix},\\
\Sigma_{03i} &= \begin{bmatrix} \begin{matrix}(d_{12}-1)(R_2+R_3)\\ +Sym\{-L_{24}-L_{34}\}\end{matrix} & -L_{25}^T-L_{35}^T & A_{d0i}^TS^T-L_{26}^T-L_{36}^T\\ * & -D_{3i}-D_{3i}^T-\gamma & D_{1i}^TS^T\\ * & * & \hat{Z}_s\end{bmatrix},
\end{aligned}$$

$$\begin{aligned}
\Sigma_4 &= \begin{bmatrix} \bar{\tau}_1 L_{11} & \bar{\tau}_2 L_{21} & \bar{\tau}_{12} L_{31} \\ \bar{\tau}_1 L_{12} & \bar{\tau}_2 L_{22} & \bar{\tau}_{12} L_{32} \\ \bar{\tau}_1 L_{13} & \bar{\tau}_2 L_{23} & \bar{\tau}_{12} L_{33} \end{bmatrix}, \ \Sigma_5 = \begin{bmatrix} \bar{\tau}_1 L_{14} & \bar{\tau}_2 L_{24} & \bar{\tau}_{12} L_{34} \\ \bar{\tau}_1 L_{15} & \bar{\tau}_2 L_{25} & \bar{\tau}_{12} L_{35} \\ \bar{\tau}_1 L_{16} & \bar{\tau}_2 L_{26} & \bar{\tau}_{12} L_{36} \end{bmatrix}, \\
\Sigma_6 &= diag(\bar{\tau}_1 M_1, \bar{\tau}_2 M_2, \bar{\tau}_{12} M_3), \\
Q_{A0i} &= SA_{0i} + A_{0i}^T S^T + Q + R_1 + R_3 + Sym\{L_{11} + L_{31}\}, \\
\hat{L}_{21} &= -L_{21} - L_{31} + L_{14}^T + L_{34}^T, \ \hat{L}_{23} = -L_{23} - L_{33} + L_{24}^T - L_{14}^T, \\
\hat{Z}_s &= \bar{\tau}_1 Z_1 + \bar{\tau}_2 Z_2 + \bar{\tau}_{12} Z_3 - S - S^T.
\end{aligned}$$

Then the closed-loop system (10)–(11) is passive.

Proof For system (10)-(11), we define the following Lyapunov functional candidate:

$$V(t) = x(t)^T P x(t) + V_1(t) + V_2(t) + V_3(t), \tag{16}$$

where

$$\begin{aligned}
V_1(t) &= \int_{t-\tau}^{t} x(s)^T Q x(s) ds, \\
V_2(t) &= \int_{t-\tau_1(t)}^{t} x(s)^T R_1 x(s) ds + \int_{t-\tau_1(t)-\tau_2(t)}^{t-\tau_1(t)} x(s)^T R_2 x(s) ds \\
&\quad + \int_{t-\tau_1(t)-\tau_2(t)}^{t} x(s)^T R_3 x(s) ds, \\
V_3(t) &= \int_{t-\bar{\tau}_1}^{t} d\theta \int_{\theta}^{t} \dot{x}(s)^T Z_1 \dot{x}(s) ds + \int_{t-\bar{\tau}_{12}}^{t-\bar{\tau}_1} d\theta \int_{\theta}^{t} \dot{x}(s)^T Z_2 \dot{x}(s) ds \\
&\quad + \int_{t-\bar{\tau}_{12}}^{t} d\theta \int_{\theta}^{t} \dot{x}(s)^T Z_3 \dot{x}(s) ds.
\end{aligned}$$

The time derivative of $V(t)$ is given by

$$\dot{V}(t) = 2x(t)^T P \dot{x}(t) + \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t),$$

where

$$\begin{aligned}
\dot{V}_1(t) &= x(t)^T Q x(t) - x(t-\tau)^T Q x(t-\tau), && (17) \\
\dot{V}_2(t) &= x(t)^T (R_1 + R_3) x(t) - (1 - \dot{\tau}_1(t)) x(t-\tau_1(t))^T (R_1 - R_2) x(t-\tau_1(t)) \\
&\quad -(1 - \dot{\tau}_1(t) - \dot{\tau}_2(t)) x(t-\tau_1(t)-\tau_2(t))^T (R_2 + R_3) x(t-\tau_1(t)-\tau_2(t)) \\
&\leq x(t)^T (R_1 + R_3) x(t) - x(t-\tau_1(t))^T [(1-d_1) R_1 - R_2] x(t-\tau_1(t)) \\
&\quad -(1-d_{12}) x(t-\tau_1(t)-\tau_2(t))^T (R_2 + R_3) x(t-\tau_1(t)-\tau_2(t)), && (18) \\
\dot{V}_3(t) &= \dot{x}(t)^T (\bar{\tau}_1 Z_1 + \bar{\tau}_2 Z_2 + \bar{\tau}_{12} Z_3) \dot{x}(t) - \int_{t-\bar{\tau}_1}^{t} \dot{x}(s)^T Z_1 \dot{x}(s) ds \\
&\quad - \int_{t-\bar{\tau}_{12}}^{t-\bar{\tau}_1} \dot{x}(s)^T Z_2 \dot{x}(s) ds - \int_{t-\bar{\tau}_{12}}^{t} \dot{x}(s)^T Z_3 \dot{x}(s) ds \\
&\leq \dot{x}(t)^T (\bar{\tau}_1 Z_1 + \bar{\tau}_2 Z_2 + \bar{\tau}_{12} Z_3) \dot{x}(t) - \int_{t-\tau_1(t)}^{t} \dot{x}(s)^T Z_1 \dot{x}(s) ds \\
&\quad - \int_{t-\tau_1(t)-\tau_2(t)}^{t-\tau_1(t)} \dot{x}(s)^T Z_2 \dot{x}(s) ds - \int_{t-\tau_1(t)-\tau_2(t)}^{t} \dot{x}(s)^T Z_3 \dot{x}(s) ds. && (19)
\end{aligned}$$

By the Newton–Leibniz formula, for any appropriately dimensioned matrices $L_i,\ i = 1,2,3$, we have the following equations

$$
\begin{aligned}
\Lambda_1 &= 2\xi(t)^T L_1[x(t) - x(t-\tau_1(t)) - \int_{t-\tau_1(t)}^{t} \dot{x}(s)ds] = 0, &(20)\\
\Lambda_2 &= 2\xi(t)^T L_2[x(t-\tau_1(t)) - x(t-\tau_1(t)-\tau_2(t)) - \int_{t-\tau_1(t)-\tau_2(t)}^{t-\tau_1(t)} \dot{x}(s)ds] = 0, &(21)\\
\Lambda_3 &= 2\xi(t)^T L_3[x(t) - x(t-\tau_1(t)-\tau_2(t)) - \int_{t-\tau_1(t)-\tau_2(t)}^{t} \dot{x}(s)ds] = 0, &(22)
\end{aligned}
$$

where

$$
\xi(t) = [x(t)^T \quad x(t-\tau)^T \quad x(t-\tau_1(t))^T \quad x(t-\tau_{12}(t))^T \quad w(t)^T \quad \dot{x}(t)^T]^T,
$$

On the other hand, for matrices $Z_j = Z_j^T,\ j = 1,2,3,\ M_i = M_i^T,\ i = 1,2,3$, which satisfy

$$
M_1 < Z_1,\ M_2 < Z_2,\ M_3 < Z_3,
$$

one can get the following inequalities:

$$
\begin{aligned}
\Upsilon_1 &= \tau_1(t)\xi(t)^T L_1 M_1^{-1} L_1^T \xi(t) - \int_{t-\tau_1(t)}^{t} \xi(t)^T L_1 Z_1^{-1} L_1^T \xi(t) ds > 0, &(23)\\
\Upsilon_2 &= \tau_1(t)\xi(t)^T L_1 M_1^{-1} L_1^T \xi(t) - \int_{t-\tau_1(t)}^{t} \xi(t)^T L_1 Z_1^{-1} L_1^T \xi(t) ds > 0, &(24)\\
\Upsilon_3 &= \tau_1(t)\xi(t)^T L_1 M_1^{-1} L_1^T \xi(t) - \int_{t-\tau_1(t)}^{t} \xi(t)^T L_1 Z_1^{-1} L_1^T \xi(t) ds > 0. &(25)
\end{aligned}
$$

It then follows from (17)-(25) that

$$
\begin{aligned}
\dot{V}(t) \le\ & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(s(t))h_j(s(t))h_l(s(t))\{\xi^T(t)\Theta_{ijl}\xi(t) + \bar{\tau}_1\xi^T(t)L_1M_1^{-1}L_1^T\xi(t)\\
& + \bar{\tau}_2\xi^T(t)L_2M_2^{-1}L_2^T\xi(t) + \bar{\tau}_{12}\xi^T(t)L_3M_3^{-1}L_3^T\xi(t)\\
& - \int_{t-\tau_1(t)}^{t} [\xi^T(t)L_1 + \dot{x}(s)^T Z_1]Z_1^{-1}[L_1^T\xi(t) + Z_1\dot{x}(s)]ds\\
& - \int_{t-\tau_{12}(t)}^{t-\tau_1(t)} [\xi^T(t)L_2 + \dot{x}(s)^T Z_2]Z_2^{-1}[L_2^T\xi(t) + Z_2\dot{x}(s)]ds\\
& - \int_{t-\tau_{12}(t)}^{t} [\xi^T(t)L_3 + \dot{x}(s)^T Z_3]Z_3^{-1}[L_3^T\xi(t) + Z_3\dot{x}(s)]ds, \qquad (26)
\end{aligned}
$$

where

$$\Theta_{ijl} = \begin{bmatrix} \Sigma_{1ijl} & \Sigma_{2ijl} \\ * & \Sigma_{3i} \end{bmatrix}, \ \bar{S} = [S^T \ 0 \ \cdots \ 0 \ S^T]^T,$$

$$\Sigma_{1ijl} = \begin{bmatrix} Q_{Ai} & SB_iK_jC_l + L_{12}^T + L_{32}^T & L_{21} - L_{11} + L_{13}^T + L_{33}^T \\ * & -Q & L_{22} - L_{12} \\ * & * & (d_1 - 1)R_1 + R_2 + Sym\{L_{23} - L_{13}\} \end{bmatrix},$$

$$\Sigma_{2ijl} = \begin{bmatrix} SA_{di} + \hat{L}_{21} & SD_{1i} + L_{15}^T + L_{35}^T & P - S + A_i^T S^T + L_{16}^T + L_{36}^T \\ -L_{22} - L_{32} & 0 & C_l^T K_j^T B_i^T S^T \\ \hat{L}_{23} & L_{25}^T - L_{15}^T & L_{26}^T - L_{16}^T \end{bmatrix},$$

$$\Sigma_{3i} = \begin{bmatrix} \begin{array}{c} (d_{12} - 1)(R_2 + R_3) \\ +Sym\{-L_{24} - L_{34}\} \end{array} & -L_{25}^T - L_{35}^T & A_{di}^T S^T - L_{26}^T - L_{36}^T \\ * & 0 & D_{1i}^T S^T \\ * & * & \hat{Z}_s \end{bmatrix}.$$

Since $Z_j > 0, \ j = 1, 2, 3$, the last three terms in (26) are all less than 0. From this, one can obtain

$$\begin{aligned} &\dot{V}(t) - 2z(t)^T\omega(t) - \gamma\omega(t)^T\omega(t) \\ \leq\ & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(s(t))h_j(s(t))h_l(s(t))\{\xi^T(t)\Phi_{ijl}\xi(t)\}, \end{aligned} \tag{27}$$

where

$$\Phi_{ijl} = \begin{bmatrix} \Sigma_{1ijl} & \hat{\Sigma}_{2ijl} & \Sigma_4 \\ * & \hat{\Sigma}_{2ijl} & \Sigma_5 \\ * & * & -\Sigma_6 \end{bmatrix},$$

$$\hat{\Sigma}_{2ijl} = \begin{bmatrix} SA_{di} + \hat{L}_{21} & SD_{1i} - E_i^T + L_{15}^T + L_{35}^T & P - S + A_i^T S^T + L_{16}^T + L_{36}^T \\ -L_{22} - L_{32} & -C_l^T K_j^T E_{1i}^T & C_l^T K_j^T B_i^T S^T \\ \hat{L}_{23} & L_{25}^T - L_{15}^T & L_{26}^T - L_{16}^T \end{bmatrix},$$

$$\hat{\Sigma}_{3i} = \begin{bmatrix} \begin{array}{c} (d_{12} - 1)(R_2 + R_3) \\ +Sym\{-L_{24} - L_{34}\} \end{array} & -L_{25}^T - L_{35}^T & A_{di}^T S^T - L_{26}^T - L_{36}^T \\ * & -D_{3i} - D_{3i}^T - \gamma & D_{1i}^T S^T \\ * & * & +\hat{Z}_s \end{bmatrix}.$$

Replacing A_i, A_{di}, B_i, in Φ_{ijl} of the inequality in (27) with $A_i = A_{0i} + \sum_{p,q=1}^{n} e_p \left|g_{a_i}^{pq}\right| e_q^T$, $A_{di} = A_{d0i} + \sum_{p,q=1}^{n} e_p \left|g_{a_{di}}^{pq}\right| e_q^T$, and $B_i = B_{0i} + \sum_{p=1}^{n}\sum_{k=1}^{n_B} e_p \left|g_{b_i}^{pk}\right| e_k^T$, respectively, we have

$$\begin{aligned} &\dot{V}(t) - 2z(t)^T\omega(t) - \gamma\omega(t)^T\omega(t) \\ \leq\ & \sum_{i=1}^{r}\sum_{j=1}^{r}\sum_{l=1}^{r} h_i(s(t))h_j(s(t))h_l(s(t))\{\xi^T(t)\Psi_{ijl}\xi(t)\} \\ =\ & \sum_{i=1}^{r}\sum_{l=1}^{r} h_i^2(s(t))h_l(s(t))\xi^T(t)\Psi_{iil}\xi(t) \\ & +2\sum_{i=1,i<j}^{r}\sum_{l=1}^{r} h_i(s(t))h_j(s(t))h_l(s(t))\xi(t)^T\frac{\Psi_{ijl} + \Psi_{jil}}{2}\xi(t). \end{aligned}$$

From (13) and (14), we can obtain that $\Psi_{iil} < 0,\ \Psi_{ijl} + \Psi_{jil} < 0$. Then we can get (12). This completes the proof. □

2.2 Delayed static output-feedback controller design

Now, we are in a position to present a solution to the passive delayed static output feedback controller design problem.

Theorem 2.2 *Consider the closed-loop fuzzy system in (5)–(7) with interval parameter uncertainties and additive delays. Given positive scalars $d_1,\ d_{12},\ \bar{\tau}_1,\ \bar{\tau}_2$ and $\bar{\tau}_{12}$, and let $\gamma > 0$ be a prescribed constant scalar. The passive control problem is solvable if there exist matrices $\tilde{P} > 0,\ \tilde{Q} > 0,\ \tilde{R}_i > 0,\ \tilde{Z}_i > 0,\ \tilde{M}_i > 0,\ X$, and positive scalars $\hat{\varepsilon}_{1ijpq},\ \hat{\varepsilon}_{2ijpq},\ \hat{\varepsilon}_{3ijpq},\ \hat{\varepsilon}_{4ijpq},\ \hat{\varepsilon}_{5ijpk},\ \hat{\varepsilon}_{6ijpk}$, for $p, q = 1, \ldots, n$ and $k = 1, \ldots, n_B$, such that the following LMIs hold:*

$$\begin{aligned} \mathcal{J}_{iil} &< 0,\ i, l = 1, \ldots, r, &(28)\\ \mathcal{J}_{ijl} + \mathcal{J}_{jil} &< 0,\ 1 \le i < j \le r,\ l = 1, \ldots r, &(29)\\ \tilde{M}_k &< \tilde{Z}_k,\ k = 1, 2, 3, &(30) \end{aligned}$$

where

$$\mathcal{J}_{ijl} = \begin{bmatrix} \mathcal{H}_{ijl} & \Omega_{eq1} & \Omega_{eq2} & \hat{W}_{eq1}^T & \hat{W}_{eq2}^T & \Omega_{ek1} & \hat{W}_{ek1}^T \\ * & -\hat{\Omega}_{\varepsilon 1ij} & 0 & 0 & 0 & 0 & 0 \\ * & * & -\hat{\Omega}_{\varepsilon 2ij} & 0 & 0 & 0 & 0 \\ * & * & * & -\hat{\Omega}_{\varepsilon 3ij} & 0 & 0 & 0 \\ * & * & * & * & -\hat{\Omega}_{\varepsilon 4ij} & 0 & 0 \\ * & * & * & * & * & -\hat{\Omega}_{\varepsilon 5ijB} & 0 \\ * & * & * & * & * & * & -\hat{\Omega}_{\varepsilon 6ijB} \end{bmatrix},$$

$$\begin{aligned} \mathcal{H}_{ijl} &= \mathcal{F}_{0ijl} + \sum_{p,q=1}^{n} \Big[(\hat{\varepsilon}_{1ijpq} + \hat{\varepsilon}_{2ijpq}) \bar{e}_p \bar{e}_p^T + (\varepsilon_{3ijpq} \Delta_{a_i}^{pq^2} + \varepsilon_{4ijpq} \Delta_{a_{di}}^{pq^2}) \hat{e}_p^T \hat{e}_p \Big] \\ &\quad + \sum_{p=1}^{n} \sum_{k=1}^{n_B} \Big(\varepsilon_{5ijpk} \bar{e}_p \bar{e}_p^T + \varepsilon_{6ijpk} \Delta_{b_i}^{pk^2} \hat{e}_p^T \hat{e}_p \Big), \\ \hat{W}_{eq1} &= \begin{bmatrix} e_q^T X & 0_{1,(m+3)n} \end{bmatrix},\ \hat{W}_{ek1} = \begin{bmatrix} 0_{1,n} & e_k^T N_j C_l & 0_{1,(m+2)n} \end{bmatrix}, \\ \hat{W}_{eq2} &= \begin{bmatrix} 0_{1,3n} & e_q^T X & 0_{1,mn} \end{bmatrix}, \Omega_{eq1} = \begin{bmatrix} \Delta_{a_i}^{11} \hat{W}_{eq1}^T & \cdots & \Delta_{a_i}^{nn} \hat{W}_{eq1}^T \end{bmatrix}, \\ \Omega_{eq2} &= \begin{bmatrix} \Delta_{a_{di}}^{11} \hat{W}_{eq2}^T & \cdots & \Delta_{a_{di}}^{nn} \hat{W}_{eq2}^T \end{bmatrix},\ \Omega_{ek1} = \begin{bmatrix} \Delta_{b_i}^{11} \hat{W}_{ek1}^T & \cdots & \Delta_{b_i}^{nn} \hat{W}_{ek1}^T \end{bmatrix}, \\ \bar{e}_p &= \begin{bmatrix} e_p \\ 0_{(m+3)n,1} \end{bmatrix},\ \hat{e}_p = \begin{bmatrix} 0_{1,mn} & e_p^T & 0_{1,3n} \end{bmatrix}, \\ \hat{\Omega}_{\varepsilon nij} &= \begin{bmatrix} \hat{\varepsilon}_{nij11} & 0 & 0 \\ * & \ddots & 0 \\ * & * & \hat{\varepsilon}_{nijnn} \end{bmatrix},\ \hat{\Omega}_{\varepsilon mijB} = \begin{bmatrix} \hat{\varepsilon}_{mij11} & 0 & 0 \\ * & \ddots & 0 \\ * & * & \hat{\varepsilon}_{mijnn_B} \end{bmatrix}, \\ \mathcal{F}_{0ijl} &= \begin{bmatrix} \mathcal{G}_{01ijl} & \mathcal{G}_{02ijl} & \mathcal{G}_4 \\ * & \mathcal{G}_{03i} & \mathcal{G}_5 \\ * & * & -\mathcal{G}_6 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned}
\mathcal{G}_{01ijl} &= \begin{bmatrix} \tilde{Q}_{A0i} & B_{0i}N_jC_l + \tilde{L}_{12}^T + \tilde{L}_{32}^T & \tilde{L}_{21} - \tilde{L}_{11} + \tilde{L}_{13}^T + \tilde{L}_{33}^T \\ * & -\tilde{Q} & \tilde{L}_{22} - \tilde{L}_{12} \\ * & * & (d_1-1)\tilde{R}_1 + \tilde{R}_2 + Sym\{\tilde{L}_{23} - \tilde{L}_{13}\} \end{bmatrix}, \\
\mathcal{G}_{02ijl} &= \begin{bmatrix} A_{d0i}X + \vec{L}_{21} & D_{1i} - X^TE_i^T + \tilde{L}_{15}^T + \tilde{L}_{35}^T & \tilde{P} - X + X^TA_{0i}^T + \tilde{L}_{16}^T + \tilde{L}_{36}^T \\ -\tilde{L}_{22} - \tilde{L}_{32} & -C_l^TN_j^TE_{1i}^T & C_l^TN_j^TB_{0i}^T \\ \vec{L}_{23} & \tilde{L}_{25}^T - \tilde{L}_{15}^T & \tilde{L}_{26}^T - \tilde{L}_{16}^T \end{bmatrix}, \\
\mathcal{G}_{03i} &= \begin{bmatrix} (d_{12}-1)(\tilde{R}_2 + \tilde{R}_3) & & \\ +Sym\{-\tilde{L}_{24} - \tilde{L}_{34}\} & -\tilde{L}_{25}^T - \tilde{L}_{35}^T & X^TA_{d0i}^T - \tilde{L}_{26}^T - \tilde{L}_{36}^T \\ * & -D_{3i} - D_{3i}^T - \gamma & D_{1i}^T \\ * & * & \vec{Z}_x \end{bmatrix}, \\
\mathcal{G}_4 &= \begin{bmatrix} \bar{\tau}_1L_{11} & \bar{\tau}_2L_{21} & \bar{\tau}_{12}L_{31} \\ \bar{\tau}_1L_{12} & \bar{\tau}_2L_{22} & \bar{\tau}_{12}L_{32} \\ \bar{\tau}_1L_{13} & \bar{\tau}_2L_{23} & \bar{\tau}_{12}L_{33} \end{bmatrix}, \ \mathcal{G}_5 = \begin{bmatrix} \bar{\tau}_1\tilde{L}_{14} & \bar{\tau}_2\tilde{L}_{24} & \bar{\tau}_{12}\tilde{L}_{34} \\ \bar{\tau}_1\tilde{L}_{15} & \bar{\tau}_2\tilde{L}_{25} & \bar{\tau}_{12}\tilde{L}_{35} \\ \bar{\tau}_1\tilde{L}_{16} & \bar{\tau}_2\tilde{L}_{26} & \bar{\tau}_{12}\tilde{L}_{36} \end{bmatrix}, \\
\mathcal{G}_6 &= diag(\bar{\tau}_1\tilde{M}_1, \bar{\tau}_2\tilde{M}_2, \bar{\tau}_{12}\tilde{M}_3), \\
\tilde{Q}_{A0i} &= A_{0i}X + X^TA_{0i}^T + \tilde{Q} + \tilde{R}_1 + \tilde{R}_3 + Sym\{\tilde{L}_{11} + \tilde{L}_{31}\}, \\
\vec{L}_{21} &= -\tilde{L}_{21} - \tilde{L}_{31} + \tilde{L}_{14}^T + \tilde{L}_{34}^T, \\
\vec{L}_{23} &= -\tilde{L}_{23} - \tilde{L}_{33} + \tilde{L}_{24}^T - \tilde{L}_{14}^T, \\
\vec{Z}_x &= \bar{\tau}_1\tilde{Z}_1 + \bar{\tau}_2\tilde{Z}_2 + \bar{\tau}_{12}\tilde{Z}_3 - X - X^T,
\end{aligned}$$

and the following equality constraint satisfied

$$MC_l = C_lX. \tag{31}$$

Furthermore, a desired passive delayed static output feedback controller is given in the form (9) with parameters as follows:

$$K_i = N_iM^{-1}, \ 1 \le i \le r. \tag{32}$$

Proof Suppose there exist matrices $\tilde{Q},\ \tilde{P},\ \tilde{R}_i,\ \tilde{Z}_j,\ \tilde{M}_i,\ i,j = 1,2,3,$ and X satisfying (28)-(31). Applying the Schur complement formula to (28) results in

$$\begin{aligned}
&\mathcal{F}_{0ijl} + \sum_{p,q=1}^{n}\Big[(\hat{\varepsilon}_{1ijpq} + \hat{\varepsilon}_{2ijpq})\bar{e}_p\bar{e}_p^T + (\varepsilon_{3ijpq}\Delta_{a_i}^{pq^2} + \varepsilon_{4ijpq}\Delta_{a_{di}}^{pq^2})\hat{e}_p^T\hat{e}_p \\
&+\hat{W}_{eq1}^T\hat{\Omega}_{\varepsilon1ij}^{-1}\Delta_{a_i}^{pq^2}\hat{W}_{eq1} + \hat{W}_{eq2}^T\hat{\Omega}_{\varepsilon2ij}^{-1}\Delta_{a_{di}}^{pq^2}\hat{W}_{eq2} + \hat{W}_{eq1}^T\hat{\Omega}_{\varepsilon3ij}^{-1}\hat{W}_{eq1} + \hat{W}_{eq2}^T\hat{\Omega}_{\varepsilon4ij}^{-1}\hat{W}_{eq2}\Big] \\
&+\sum_{p=1}^{n}\sum_{k=1}^{n_B}(\varepsilon_{5ijpk}\bar{e}_p\bar{e}_p^T + \varepsilon_{6ijpk}\Delta_{b_i}^{pk^2}\hat{e}_p^T\hat{e}_p + \hat{W}_{ek1}^T\hat{\Omega}_{\varepsilon5ijB}^{-1}\Delta_{b_i}^{pk^2}\hat{W}_{ek1} \\
&+\hat{W}_{ek1}^T\hat{\Omega}_{\varepsilon6ijB}^{-1}\hat{W}_{ek1}) < 0,
\end{aligned}$$

then, by Lemma 2.1, it is easy to have

$$\begin{aligned}
\mathcal{F}_{0ijl} \quad &+ \quad sym\{\sum_{p,q=1}^{n}[\bar{e}_p \mid f_{A_i}^{pq} \mid \hat{W}_{eq1} + \bar{e}_p \mid f_{A_{di}}^{pq} \mid \hat{W}_{eq2} + \hat{W}_{eq1}^T \mid f_{A_i}^{pq} \mid^T \hat{e}_p \\
&+ \quad \hat{W}_{eq2}^T \mid f_{A_{di}}^{pq} \mid^T \hat{e}_p] + \sum_{p=1}^{n}\sum_{k=1}^{n_B}[\bar{e}_p \mid f_{B_i}^{pk} \mid \hat{W}_{ek1} + \hat{W}_{ek1}^T \mid f_{B_i}^{pk} \mid^T \hat{e}_p]\} < 0.
\end{aligned}$$

Replacing $A_i = A_{0i} + \sum_{p,q=1}^{n} e_p \left|g_{a_i}^{pq}\right| e_q^T$, $A_{di} = A_{d0i} + \sum_{p,q=1}^{n} e_p \left|g_{a_{di}}^{pq}\right| e_q^T$, and $B_i = B_{0i} + \sum_{p=1}^{n}\sum_{k=1}^{n_B} e_p \left|g_{b_i}^{pk}\right| e_k^T$, in the proceeding inequality with A_i, A_{di}, B_i, respectively, we can get

$$\begin{bmatrix} \mathcal{G}_{1ijl} & \mathcal{G}_{2ijl} & \mathcal{G}_4 \\ * & \mathcal{G}_{3i} & \mathcal{G}_5 \\ * & * & -\mathcal{G}_6 \end{bmatrix} < 0, \tag{33}$$

where

$$\begin{aligned}
\mathcal{G}_{1ijl} &= \begin{bmatrix} \tilde{Q}_{Ai} & B_iN_jC_l + \tilde{L}_{12}^T + \tilde{L}_{32}^T & \tilde{L}_{21} - \tilde{L}_{11} + \tilde{L}_{13}^T + \tilde{L}_{33}^T \\ * & -\tilde{Q} & \tilde{L}_{22} - \tilde{L}_{12} \\ * & * & (d_1-1)\tilde{R}_1 + \tilde{R}_2 + Sym\{\tilde{L}_{23} - \tilde{L}_{13}\} \end{bmatrix}, \\
\mathcal{G}_{2ijl} &= \begin{bmatrix} A_{di}X + \vec{L}_{21} & D_{1i} - X^TE_i^T + \tilde{L}_{15}^T + \tilde{L}_{35}^T & \tilde{P} - X + X^TA_i^T + \tilde{L}_{16}^T + \tilde{L}_{36}^T \\ -\tilde{L}_{22} - \tilde{L}_{32} & -C_l^TN_j^TE_{1i}^T & C_l^TN_j^TB_{0i}^T \\ \vec{L}_{23} & \tilde{L}_{25}^T - \tilde{L}_{15}^T & \tilde{L}_{26}^T - \tilde{L}_{16}^T \end{bmatrix}, \\
\mathcal{G}_{3i} &= \begin{bmatrix} \begin{array}{c}(d_{12}-1)(\tilde{R}_2 + \tilde{R}_3) \\ +Sym\{-\tilde{L}_{24} - \tilde{L}_{34}\}\end{array} & -\tilde{L}_{25}^T - \tilde{L}_{35}^T & X^TA_{di}^T - \tilde{L}_{26}^T - \tilde{L}_{36}^T \\ * & -D_{3i} - D_{3i}^T - \gamma & D_{1i}^T \\ * & * & \vec{Z}_x \end{bmatrix}.
\end{aligned}$$

Suppose there exits a nonsingular matrix S satisfying

$$S = X^{-T}.$$

Without loss of generality, we can define

$$\begin{aligned}
\tilde{L}_{1j} &= X^TL_{1j}X,\ \tilde{L}_{2j} = X^TL_{2j}X,\ \tilde{L}_{3j} = X^TL_{3j}X,\ j = 1,2,3,4,6, \\
\tilde{Q} &= X^TQX,\ \tilde{P} = X^TPX,\ \tilde{R}_i = X^TR_iX,\ \tilde{Z}_j = X^TZ_jX,\ \tilde{M}_i = X^TM_iX, \\
\tilde{L}_{i5} &= L_{i5}X,\ i,j = 1,2,3.
\end{aligned}$$

Now pre- and post- multiplying the LMIs in (33) by $diag(S,S,S,S,I,S,\cdots,S)$ and $diag(S^T,S^T,S^T,S^T,I,\ S^T,\cdots,S^T)$, respectively, then we have $\Phi_{ijl} < 0$, which, by Schur complement can be converted to $\Psi_{ijl} < 0$. Following the similar procedure, we can get $\Psi_{ijl} + \Psi_{jil} < 0$ from the inequality in (29). Pre- and post- multiplying the LMI in (30) with S and S^T, we can get (15). Thus, we obtain (13)-(15) in Theorem 2.1. Finally, by Theorem 2.1, the closed-loop system in (10)-(11) is passive. The proof is completed. $\square$

Remark 2.1 It is observed from Theorem 2.2 that the static output feedback controller design is the feasibility problem of LMIs (28)-(30) with equality constraint (31). However, this kind of problem has been solved in [19] via genetic algorithms and in [18] via the LMI-based algorithms, which can be easily implemented with polynomial running time. Hence, in this paper, we will convert the equality constraint problem to the LMI problem [4].

3 An Illustrative Example

In this section, we provide an example to illustrate the passive delayed static output feedback controller design approach developed in this paper.

The uncertain Takagi–Sugeno (T-S) fuzzy system considered in this example is with two rules with the following parameters:

$$\begin{aligned}
A_{01} &= \begin{bmatrix} -5 & 0.2 \\ 0 & 0.01 \end{bmatrix}, \ A_{d01} = \begin{bmatrix} -0.1 & 0 \\ 0.1 & -0.1 \end{bmatrix}, \ B_{01} = \begin{bmatrix} 1 & 3 \\ 2 & 1 \end{bmatrix}, \\
C_1 &= \begin{bmatrix} 0.2 & 0.1 \\ 0.1 & 1 \end{bmatrix}, \ D_{11} = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix}, \ D_{31} = 0.2, \ E_1 = \begin{bmatrix} -0.4 & 0.1 \end{bmatrix}, \\
E_{11} &= \begin{bmatrix} 0.1 & 0.2 \end{bmatrix}, \ A_{02} = \begin{bmatrix} -6 & 0 \\ 0.1 & 0.05 \end{bmatrix}, \ A_{d02} = \begin{bmatrix} -0.2 & 0.1 \\ 0.1 & -0.2 \end{bmatrix}, \\
B_{02} &= \begin{bmatrix} 1 & 4 \\ 2 & 4 \end{bmatrix}, \ C_2 = C_1, \ D_{12} = D_{11}, \ D_{32} = 0.3, \ E_2 = \begin{bmatrix} 0.1 & 0.4 \end{bmatrix}, \\
E_{12} &= \begin{bmatrix} 0.2 & 0.1 \end{bmatrix}, \ \Delta a_i^{pq} = 0.001I, \ \Delta a_{di}^{pq} = 0.002I, \ \Delta b_i^{pk} = 0.001I.
\end{aligned}$$

The membership functions are chosen as:

$$h_1(x_1(t)) = \begin{cases} \frac{1}{3}, & for\ x_1 < -1, \\ \frac{2}{3} + \frac{1}{3}x_1, & for\ |x_1| \le 1, \\ 1, & for\ x_1 > 1. \end{cases} \quad h_2(x_1(t)) = 1 - h_1(x_1(t)).$$

In this example, given $\tau = 0.1,\ \bar{\tau}_1 = 0.4$, we have the maximum of $\bar{\tau}_2 = 3$; while given $\tau = 0.1,\ \bar{\tau}_2 = 0.1$, we have the maximum of $\bar{\tau}_1 = 3$.

In order to design a fuzzy passive static output feedback controller for the T-S model, we first choose

$$\tau = 0.1,\ \bar{\tau}_1 = 0.1,\ \bar{\tau}_2 = 0.1,\ d_1 = 0.4,\ d_{12} = 0.8,\ \gamma = 0.5$$

and the initial condition is $x(0) = \begin{bmatrix} 0.1 & -0.8 \end{bmatrix}^T$, the disturbance input $w(t)$ is assumed to be

$$w(t) = \frac{1}{t + 0.1},\ t \ge 0.$$

Then, solving the LMIs in (28)–(30) and (31), we obtain the solution as follows:

$$N_1 = \begin{bmatrix} 0.1060 & -0.0538 \\ -0.0488 & 0.0746 \end{bmatrix}, \ N_2 = \begin{bmatrix} 0.4842 & -0.2166 \\ -0.1829 & 0.1025 \end{bmatrix},$$

and the fuzzy delayed static output feedback controller gains are given by

$$K_1 = \begin{bmatrix} 2.2558 & -0.3342 \\ -0.6589 & 0.2232 \end{bmatrix}, \ K_2 = \begin{bmatrix} 10.8439 & -2.4939 \\ -3.9293 & 0.9533 \end{bmatrix}.$$

With the static output feedback fuzzy controller, the simulation result of the state response of the nonlinear system are given in Figure 1. From the simulation result, it can be seen the designed fuzzy output feedback controller is effective.

4 Conclusion

The problem of passive delayed static output feedback control for uncertain Takagi–Sugeno fuzzy systems with interval parameters and additive delays has been studied. In terms of linear matrix inequalities, a sufficient delay-dependent condition for the existence of a full-order fuzzy delayed static output feedback controller, which guarantees the closed-loop system is passive, has been obtained. An example has been provided to show the effectiveness of the proposed method.

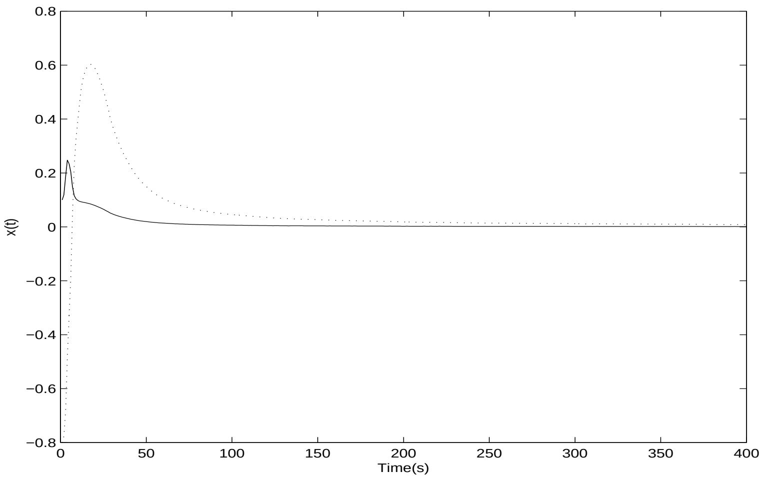

Figure 1: State response $x_1(t)$ (—) and $x_2(t)$ $(\cdots)$.

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References

[1] Chang, W. J., Ku, F. C. and Ku, C. C. Fuzzy control for discrete passive affine T-S fuzzy systems with observer feedback. *Springer Berlin* **2** (2007) 623–631.

[2] Chen, S.-S., Chang, Y.-C., Su, S.-F., Chung, S.-L. and Lee, T.-T. Robust static output-feedback stabilization for nonlinear discrete-time systems with time delay via fuzzy control approach. *IEEE Trans. Fuzzy Syst.* **13** (2005) 263–272.

[3] Dong, J. and Yang, G.-H. Static output feedback control synthesis for discrete-time T-S fuzzy systems. *International Journal of Control, Automation, and Systems* **5** (2007) 349–354.

[4] Du, H. and Zhang, N. Static output feedback control for electrohydraulic active suspensions via T-S fuzzy model approach. *ASME J. Dyn. Syst., Meas. Control* **131** (2009) 2120–2127.

[5] Fridman, E. and Shaked, U. On delay-dependent passivity. *IEEE Trans. Automat. Control* **47** (2002) 664–669.

[6] Karimi, H. and Gao, H. LMI-based H_∞ synchronization of second-order neutral master-slave systems using delayed output feedback control. *Int. J. Control, Automation and Systems* **7** (2009) 371–380.

[7] Li, Y., Fu, Y. and Duan, G. Robust passive control for T-S fuzzy systems. *Springer Berlin* **4114** (2006) 146–151.

[8] Park, J. H. and Kwon, O. On guaranteed cost control of neutral systems by retarded integral state feedback. *Appl. Math. Comput.* **165** (2005) 393–404.

[9] Psillakis, H. E. and Alexandridis, A. T. A simple nonlinear adaptive-fuzzy passivity-based control of power systems. *Nonlinear Dynamics and Systems Theory* **7** (1) (2007) 51–67.

[10] Pyragas, K. Continuous control of chaos by self-controlling feedback. *Phys. Lett. A* **170** (1992) 421–428.

[11] Ramakrishnan, K. and Ray, G. Delay-dependent stability analysis of linear system with additive time-varying delays. In: *Proceedings of 5th Annual IEEE Conference on Automation Science and Engineering.* Bangalore, India, 2009, 122–126.

[12] Song, X., Xu, S. and Shen, H. Robust H_∞ control for uncertain fuzzy systems with distributed delays via output feedback controllers. *Information Sciences* **178** (2008) 4341–4356.

[13] Takagi, T. and Sugeno, M. Fuzzy identification of systems and its applications to modeling and control. *IEEE Trans. Syst. Man Cybernet.* **15** (1985) 116–132.

[14] Wu, L. and Zheng, W. Passivity-based sliding mode control of uncertian singular time-delay systems. *Automatica* **45** (2009) 2120–2127.

[15] Xie, L. Output feedback H_∞ control of systems with parameter uncertainty. *Int. J. Control* **63** (1996) 741–750.

[16] Xu, S. andLam, J. Robust H_∞ control for uncertain discrete-time-delay fuzzy system via output feedback controllers. *IEEE Trans. Fuzzy Syst.* **13** (2005) 82–93.

[17] Zhao, Y.,Zhang, C. and Gao, H. A new approach to guaranteed cost control of T-S fuzzy dynamic systems with interval parameter uncertainties. *IEEE Trans. Syst. Man Cybernet. Part B* **39** (2009) 1516–1527.

[18] Ho, D. W. C., and Niu, Y. Robust fuzzy design for nonlinear uncertian stochastic systems via sliding-mode control. *IEEE Trans. Fuzzy Syst.* **15** (2007) 350–358.

[19] Du, H., Lam, J. and Sze, K. Y. Non-fragile output feedback H_∞ vehicle suspension control using genetic algorithm. *Eng. Applic. Artif. Intell.* **16** (2003) 667–680.