Nonlinear Dynamics and Systems Theory, **11** (2) (2011) 155–163

Self Recurrent Wavelet Neural Network Based Direct Adaptive Backstepping Control for a Class of Uncertain Non-Affine Nonlinear Systems

A. Kulkarni *, M. Sharma and S. Puntambekar

Medi-Caps Institute of Technology and Management, Indore, India

Received: March 19, 2010; Revised: March 30, 2011

Abstract: This paper proposes an adaptive backstepping control strategy for a class of uncertain non affine systems using self recurrent neural networks. To assure the stable tracking of nonlinear non affine system, it is first converted to an affine like form and subsequently a wavelet based adaptive backstepping controller is developed. Self recurrent wavelet neural network (SRWNN) is used to approximate the uncertainties present in the system as well as to compensate the highly dynamic nonlinearities inserted by these uncertainties in the control terms. In addition robust control terms are also designed to attenuate the approximation error due to SRWNN. Based on the Lyapunov theory, the online adaptation laws and stability of the closed loop system are verified. A numerical example is provided to verify the effectiveness of theoretical development.

Keywords: *non-affine systems; self recurrent wavelet networks; backstepping control; adaptive control; Lyapunov analysis.*

Mathematics Subject Classification (2000): 49J35, 34A34, 92C20.

1 Introduction

Over last few years, several efforts on the development of adaptive control strategies for uncertain nonlinear systems have been cited in the literature. In these cases the common assumption was that the system is affine in input [1, 2]. However the development of control strategies is still an active area of research.

To deal with the non affine systems, two control strategies are cited in the literature. One is based on the dynamic inversion satisfying the assumptions of Tikhonov theorem

* Corresponding author: mailto:a.kulkarni17@gmail.com

from the singular perturbation theory. Other is based on the conversion of non affine system to an affine like form by applying a suitable transformation and designing the controller for the later form by implicit function theorem [3]–[7].

Backstepping is a recursive design methodology where some appropriate functions of state variables as pseudocontrol inputs for lower dimension subsystems of the overall system are derived. Each backstepping stage results in a new pseudocontrol design, expressed in terms of the pseudocontrol designs from preceding design stages. When the procedure is terminated, a feedback design for the true control input results, which achieves the original design objective by virtue of a final Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage. Thus, the backstepping control approach is capable of keeping the robustness properties with respect to the uncertainties [13]–[15]. Via adaptive backstepping this methodology can be effectively extended to non linear systems with unmodelled dynamics [1].

Based on the concept of transformation of non affine systems into affine like form, some researchers have proposed adaptive backstepping based control schemes for non affine uncertain systems [7].

Employment of neural network (NN) as an approximation tool in adaptive control strategies has greatly relaxed the assumptions on linear parameterized nonlinearities and thereby broadens the class of the uncertain nonlinear systems which can be effectively dealt by adaptive controllers [8]. However there are certain difficulties associated with NN based controller. The basis functions are generally not orthogonal or redundant; i.e., the network representation is not unique and is probably not the most efficient one. Furthermore, the convergence of neural networks may not be guaranteed. Even when it exhibits a good convergence rate, the training procedure may still be trapped in some local minima depending on the initial settings. Wavelet neural networks are feed-forward neural networks using wavelets as activation function. Due to their space and frequency localization properties, the learning capability of WNN is superior to conventional neural networks. Training algorithms for WNN converge in smaller number of iterations than for conventional neural networks. These WNN combines the capability of artificial neural network for learning ability and capability of wavelet decomposition for identification ability. Thus WNN based control systems can achieve better control performance than NN based control systems [9, 10]. The feedforward structure of the conventional WNN limits the applicability of these networks only to static environmental conditions. These networks are not very effective under the frequently changing operating conditions and dynamic properties as they can not adapt rapidly under such circumstances. To overcome this problem, a feedback mechanism is inserted in conventional WNN giving rise to either output recurrent WNN (ORWNN) or self recurrent WNN (SRWNN). These recurrent networks combines the properties of recurrency with the convergence properties of WNN to solve the complex control problems [11, 12].

This paper deals with the designing of a backstepping based adaptive tracking controller for a class of uncertain non affine systems. SRWNN are used for approximating the system uncertainty as well as to compensate the nonlinearities arising in the controller terms due to these uncertainties.

For the class of the system under consideration the backstepping control terms contain the system nonlinearities as well as their derivatives of various orders. Consideration of these derivatives while deriving the controller terms results in numerically untraceable solution, whereas if these derivatives are neglected, it results in approximate backstepping.

In this work such derivative terms are approximated by using SRWNN, thereby reducing the mathematical complexities as well as improving the accuracy of the controller strategy.

The paper is organized as follows: Section 2 deals with the system preliminaries, system description is given in Section 3. SRWNN based backstepping controller designing aspects are discussed in Section 4. Effectiveness of the proposed strategy is illustrated through an example in Section 5 while Section 6 concludes the paper.

2 System Preliminaries

2.0.1 Self recurrent wavelet neural network

Wavelet network is a type of building block for function approximation. The building block is obtained by translating and dilating the mother wavelet function. SRWNN is modified form of WNN composed of a self feedback wavelon layer as shown in Figure 1. Due to the self feedback layer the wavelon layer can store the past information of the network, thereby capturing the dynamic response of the system. This modification allows SRWNN to approximate dynamic nonlinearities with high degree of accuracy. This makes SRWNN more suitable tool for the adaptive control strategies as compared to conventional WNN.

Output of an n dimensional SRWNN with m wavelet nodes is

$$f = \sum_{i=1}^{m} \alpha_i \varphi_i \left(\theta_i, \bar{\varphi}_i, x, w_i, c_i\right), \tag{1}$$

where φ_i is the i^{th} wavelet node given by

$$\varphi_i \left(\theta_i, \bar{\varphi}_i, x, w_i, c_i\right) = \prod_{j=1}^{n} \varphi_{ij}(\theta_{ij}, \bar{\varphi}_{ij}, x, w_{ij}, c_{ij}), \tag{2}$$

where φ_{ij} is the j^{th} wavelon of i^{th} wavelet node. $x = [x_1, x_2, \ldots, x_n]^T$ is the vector of the states of the system and act as external input vector the SRWNN, whereas $\bar{\varphi}_i = [\bar{\varphi}_{i1}, \bar{\varphi}_{i2}, \ldots, \bar{\varphi}_{in}]$ is the previous value vector of the wavelon constituting the i^{th} wavelet node This vector serves as the memory element and stores the previous information of the network, and acts as the feedback input for the respective wavelon. $\theta_i = [\theta_{i1}, \theta_{i2}, \ldots, \theta_{in}]$ is the weight vector of the feedback input. Whereas $w_i = [w_{i1}, w_{i2}, \ldots, w_{in}]$ and $c_i = [c_{i1}, c_{i2}, \ldots, c_{in}]$ are dilate and translate vectors respectively. The net input applied to the wavelet network is given by $z_i = [x_1 + \theta_{i1}\varphi'_{i1}, x_2 + \theta_{i2}\varphi'_{i2}, \ldots, x_n + \theta_{in}\varphi'_{in}]^T$.

Now (1) can be rewritten as

$$f = \alpha^T \varphi \left(x, \theta, \bar{\varphi}, w, c\right), \tag{3}$$

where $w = [w_1, w_2, ..., w_m]^T \in R^{mxn}$ and $c = [c_1, c_2, ..., c_m]^T \in R^{mxn}$ are dilation and translation parameters respectively; $\alpha = [\alpha_1, \alpha_2, .., \alpha_m]^T \in R^m$ and $\theta = [\theta_1, \theta_2, .., \theta_m]^T \in R^{nxm}$ are the output and feedback weights respectively. $\bar{\varphi} = [\bar{\varphi}_1, \bar{\varphi}_2 .., \bar{\varphi}_m]^T \in R^{nxm}$ is the feedback input vector of SRWNN.

Let f^* be the optimal function approximation using an ideal wavelet approximator then

$$f = f^* + \Delta = \alpha^{*T} \varphi^* + \Delta, \tag{4}$$

where $\varphi^* = \varphi(x, \theta^*, \bar{\varphi}, w^*, c^*)$ and $\alpha^*, w^*, c^*, \theta^*$ are the optimal parameter vectors of α, w, c, θ respectively and Δ denotes the approximation error and is assumed to be bounded by $|\Delta| \leq \Delta^*$, in which Δ^* is a positive constant. Optimal parameter vectors needed for the best approximation of the function are difficult to determine so define an estimate function as

$$\hat{f} = \hat{\alpha}^T \hat{\varphi}, \tag{5}$$

where $\hat{\varphi} = \varphi\left(x, \hat{w}, \hat{c}, \hat{\theta}, \bar{\varphi}\right)and\hat{\alpha}, \hat{w}, \hat{c}, \hat{\theta}$ are the estimates of $\alpha^*, w^*, c^*, \theta^*$ respectively. Define the estimation error as

$$\tilde{f} = f - \hat{f} = f^* - \hat{f} + \Delta = \alpha^T \tilde{\varphi} + \hat{\alpha}^T \tilde{\varphi} + \tilde{\alpha}^T \hat{\varphi} + \Delta, \tag{6}$$

where $\tilde{\alpha} = \alpha^* - \hat{\alpha}$, $\tilde{\varphi} = \varphi^* - \hat{\varphi}$.

By properly selecting the number of nodes, the estimation error $\tilde{f}$ can be made arbitrarily small on the compact set so that the bound $\left\|\tilde{f}\right\| = \tilde{f}_m$ holds for all $x \in \Re$.

Using Taylor expansion linearization technique to transform the nonlinear function into a partially linear form as a step towards the derivation of online tuning laws for the wavelet parameters to achieve the favorable estimation of system dynamics [1]

$$\tilde{\varphi} = A^T \tilde{w} + B^T \tilde{c} + C^T \tilde{\theta} + h, \tag{7}$$

where $\tilde{w} = w^* - \hat{w}, \tilde{c} = c^* - \hat{c}, \tilde{\theta} = \theta^* - \hat{\theta}$ and h are the vectors of higher order terms and

$$\begin{aligned} A &= \left[\tfrac{d\varphi_1}{dw}, \tfrac{d\varphi_2}{dw}, ..., \tfrac{d\varphi_m}{dw}\right]\Big|_{w=\hat{w}}, \\ B &= \left[\tfrac{d\varphi_1}{dc}, \tfrac{d\varphi_2}{dc}, ..., \tfrac{d\varphi_m}{dc}\right]\Big|_{c=\hat{c}}, \\ C &= \left[\tfrac{d\varphi_1}{d\theta}, \tfrac{d\varphi_2}{d\theta}, ..., \tfrac{d\varphi_m}{d\theta}\right]\Big|_{\theta=\hat{\theta}}, \end{aligned}$$

with

$$\begin{aligned} \tfrac{d\hat{\varphi}_i}{dw} &= \left[0, ..., 0, \tfrac{d\hat{\varphi}_i}{dw_{1i}}, \tfrac{d\hat{\varphi}_i}{dw_{2i}}, ..., \tfrac{d\hat{\varphi}_i}{dw_{ni}}, 0...0\right]^T, \\ \tfrac{d\hat{\varphi}_i}{dc} &= \left[0, ..., 0, \tfrac{d\hat{\varphi}_i}{dc_{1i}}, \tfrac{d\hat{\varphi}_i}{dc_{2i}}, ..., \tfrac{d\hat{\varphi}_i}{dc_{ni}}, 0...0\right]^T, \\ \tfrac{d\hat{\varphi}_i}{d\theta} &= \left[0, ..., 0, \tfrac{d\hat{\varphi}_i}{d\theta_{1i}}, \tfrac{d\hat{\varphi}_i}{d\theta_{2i}}, ..., \tfrac{d\hat{\varphi}_i}{d\theta_{ni}}, 0...0\right]^T. \end{aligned}$$

Substituting (7) into (6), we have

$$\tilde{f} = \left(\tilde{\alpha}^T \left(\hat{\varphi} - \mathrm{A}_1^T \hat{w} - B_1^T \hat{c} - C^T \hat{\theta}\right) + \tilde{w}^T A \hat{\alpha} + \tilde{c}^T B \hat{\alpha} + \tilde{\theta}^T C \hat{\alpha} + \varepsilon\right), \tag{8}$$

where ε is the uncertain term.

3 System Description

Consider a non affine system of the form

$$\begin{aligned} &\dot{x}_1 = x_2 + \phi_1(x, u), \\ &\dot{x}_2 = x_3 + \phi_2(x, u), \\ &\vdots \\ &\dot{x}_n = \phi_n(x, u), \\ &y = x_1, \end{aligned} \tag{9}$$

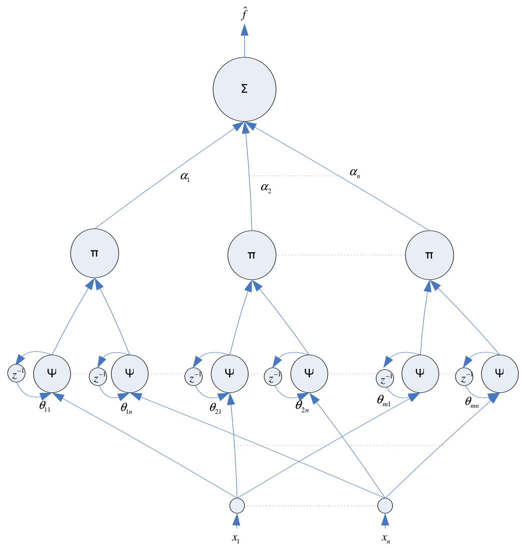

Figure 1: Self recurrent wavelet network.

where $x = [x_1, x_2, ..., x_n]^T$, u, y are state variable, control input and output respectively. $\phi = [\phi_1, \phi_2, ..., \phi_n]^T : \Re^{n+1} \to \Re^n$ are smooth unknown, nonlinear functions of state variables and input.

Applying the transformation the system (9) can be converted to an affine like form and can be rewritten as [7]

$$\begin{aligned} &\dot{x}_1 = x_2 + \phi_1(x, u), \\ &\dot{x}_2 = x_3 + \phi_2(x, u), \\ &\vdots, \\ &\dot{x}_n = \phi_n(x, u) = u + (\phi_n(x, u) - u) = u + f(x, u), \\ &y = x_1. \end{aligned} \tag{10}$$

The objective is to formulate a state feedback control law to achieve the desired tracking performance. The control law is formulated using the transformed system (10). Let $\bar{y}_d = [y_d, \dot{y}_d, \ldots, \overset{n-1}{y_d}]^T$ be the vector of desired tracking trajectory. Following assumptions are taken for the systems under consideration.

Assumption 3.1 1. Desired trajectory $y_d(t)$ is assumed to be smooth, continuous C^n and available for measurement.

2. The nonlinear function $\phi_n(x, u)$ satisfies: $\left|\frac{\partial}{\partial u}\phi_n(x, u)\right| \geq \beta \geq 0$, which ensures the controllability of the system.

In the next section the SRWNN based adaptive control strategy for (10) is discussed.

4 SRWNN Based Adaptive Backstepping Controller Design

Define the state tracking error vector $e(t)$ as $e(t) = [x_1 - y_d, x_2 - \dot{y}_d, \ldots, x_n - \overset{n-1}{y}]^T$. So the error system of (10) becomes

$$\dot{e}_1 = e_2 + \phi_1(x, u), \tag{11}$$

$$\dot{e}_2 = e_3 + \phi_2(x, u), \tag{12}$$

$$\vdots$$

$$\dot{e}_n = u + f(x, u) - \overset{n}{y}_d\,. \tag{13}$$

Considering subsystem (11), let e_{2d} be the desired value of the e_2 required to stabilize (11), $e_{2d} = -k_1 e_1 - \hat{\phi}_1 + e_{2dr}$, where $k_1 > 0$, $\hat{\phi}_1$ is the SRWNN approximation of ϕ_1. e_{2dr} is the robust term used to attenuate the uncertainties introduced by the SRWNN. The online tuning laws for the wavelet parameters are:

$$\begin{array}{l} \dot{\tilde{\alpha}}_1 = -\dot{\hat{\alpha}}_1 = \beta_{11} e_1 (\hat{\varphi}_1 - A_1^T \hat{w}_1 - B_1^T \hat{c}_1 - C_1^T \hat{\theta}_1), \\ \dot{\tilde{w}}_1 = -\dot{\hat{w}}_1 = \beta_{12} e_1 A \hat{\alpha}_1, \\ \dot{\tilde{c}}_1 = -\dot{\hat{c}}_1 = \beta_{13} e_1 B_1 \hat{\alpha}_1, \\ \dot{\tilde{\theta}}_1 = -\dot{\hat{\theta}}_1 = \beta_{14} e_1 C_1 \hat{\alpha}_1. \end{array} \tag{14}$$

And the robust control term is defined as

$$e_{2dr} = -\frac{(\rho_1^2 + 1) e_1}{2\rho_1^2}, \tag{15}$$

where ρ_1 is the prescribed attenuation, β_{11}, β_{12}, β_{13} and β_{14} are the positive learning rates. Similarly the pseudo controller design for recursive i^{th} subsystem is given by

$$e_{(i+1)d} = (-\delta_i - k_i(e_i - e_{id}) - (e_{i-1} - e_{(i-1)d}) + e_{(i+1)dr}), \tag{16}$$

where $k_i > 0$ and δ_i is the approximation of $\phi_i - \dot{e}_{id}$. The term $\dot{e}_{id}$ contains the higher order derivatives of previous pseudo controller terms which in turn consist of state variables, input and their derivatives. Presence of all such terms makes it highly dynamic in nature and hence SRWNN is the most appropriate tool for the approximation if such highly dynamic nonlinear term. $e_{(i+1)dr}$ is the robust term used to attenuate the uncertainties introduced by the SRWNN. The online tuning laws for the wavelet parameters are:

$$\begin{array}{l} \dot{\tilde{\alpha}}_i = -\dot{\hat{\alpha}}_i = \beta_{i1} (e_i - e_{id}) (\hat{\varphi}_i - A_i^T \hat{w}_i - B_i^T \hat{c}_i - C_i^T \hat{\theta}_i), \\ \dot{\tilde{w}}_i = -\dot{\hat{w}}_i = \beta_{i2} (e_i - e_{id}) A_i \hat{\alpha}_i, \\ \dot{\tilde{c}}_i = -\dot{\hat{c}}_i = \beta_{i3} (e_i - e_{id}) B_i \hat{\alpha}_i, \\ \dot{\tilde{\theta}}_i = -\dot{\hat{\theta}}_i = \beta_{i4} (e_i - e_{id}) C_i \hat{\alpha}_i. \end{array} \tag{17}$$

And the robust control term is defined as

$$e_{idr} = -\frac{(\rho_i^2 + 1)(e_i - e_{id})}{2\rho_i^2}, \tag{18}$$

where ρ_i is the prescribed attenuation, β_{i1}, β_{i2}, β_{i3} and β_{i4} are the positive learning rates. Proceeding in the same manner the control law for the overall system is defined as

$$u = (-\delta_n - k_n(e_n - e_{nd}) - (e_{n-1} - e_{(n-1)d}) + u_r + \overset{n}{y}_d), \tag{19}$$

where $k_n > 0$ and δ_n is the approximation of $f - \dot{e}_{nd}$. u_r is the robust term used to attenuate the uncertainties introduced by the SRWNN. The online tuning laws for the wavelet parameters are:

$$\begin{aligned}
\dot{\hat{\alpha}}_n &= -\dot{\tilde{\alpha}}_n = \beta_{n1}(e_n - e_{nd})(\hat{\varphi}_n - A_n^T \hat{w}_n - B_n^T \hat{c}_n - C_n^T \hat{\theta}_n),\\
\dot{\hat{w}}_n &= -\dot{\tilde{w}}_n = \beta_{n2}(e_n - e_{nd})A_n \hat{\alpha}_n,\\
\dot{\hat{c}}_n &= -\dot{\tilde{c}}_n = \beta_{n3}(e_n - e_{nd})B_n \hat{\alpha}_n,\\
\dot{\hat{\theta}}_n &= -\dot{\tilde{\theta}}_n = \beta_{n4}(e_n - e_{nd})C_n \hat{\alpha}_n.
\end{aligned} \tag{20}$$

And the robust control term is defined as

$$u_r = -\frac{(\rho_n^2 + 1)(e_n - e_{nd})}{2\rho_n^2}, \tag{21}$$

where ρ_n is the prescribed attenuation, β_{n1}, β_{n2}, β_{n3} and β_{n4} are the positive learning rates.

5 Simulation Results

Simulation is performed to verify the effectiveness of proposed SRWNN based backstepping control strategy. Consider a system of the form

$$\begin{aligned}
&\dot{x}_1 = x_2 + 0.1x_1^2,\\
&\dot{x}_2 = \tfrac{u^3}{3} + \sin u + ux_1^2 + 0.5x_1^4,\\
&y = x_1.
\end{aligned} \tag{22}$$

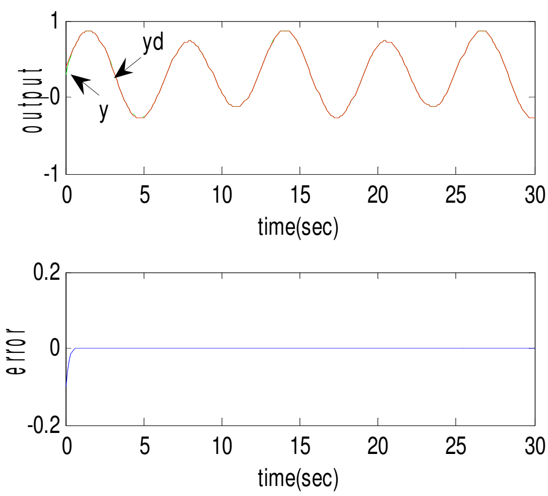

Figure 2: System output and tracking error.

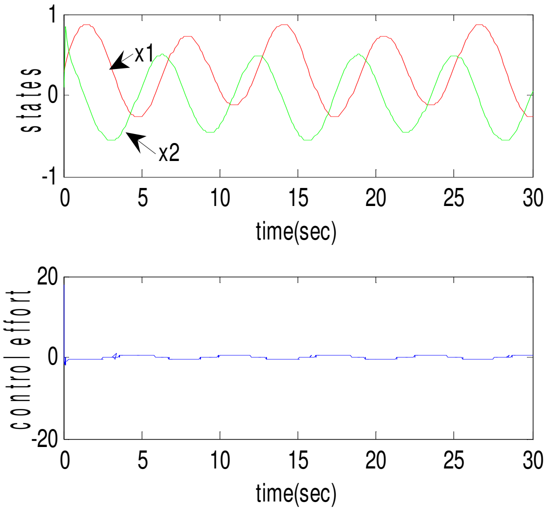

Figure 3: States of the system and control signal.

System belongs to the class of uncertain non affine systems defined by (9) with $n = 2$. The proposed controller strategy is applied to this system with an objective to solve the tracking problem of system.

The desired trajectory is taken as $y_d = 0.5 \sin t + 0.1 \cos \frac{t}{2} + 0.3$. Initial conditions are taken as $[0.3, 0.3]^T$. Attenuation level for the robust control terms is taken as 0.01. Controller parameters are taken as $k_1 = 10, k_2 = 10$. Two self recurrent wavelet networks with Mexican hat as the mother wavelet are used for approximating the unknown system dynamics. Wavelet parameters for these wavelet networks are tuned online using the proposed adaptation laws, initial conditions for all the wavelet parameters are set to zero. Simulation results are shown in Figure 2 and Figure 3. As observed from the figures, system response tracks the desired trajectory rapidly.

6 Conclusion

A SRWNN based adaptive backstepping control strategy is proposed for solving the tracking control problem for a class of non affine systems with unknown system dynamics. Self recurrent adaptive wavelet networks are used for approximating the unknown system dynamics of the system. Adaptation laws are developed for online tuning of the wavelet parameters. The theoretical analysis is validated by the simulation results.

References

[1] Hsu, C.F., Lin, C.M. and Lee T.T. Wavelet adaptive backstepping control for a class of nonlinear system. *IEEE Transactions on Neural Networks* **17** (5) (2006) 1175–1183.

[2] Wai, R.J. Chang. Backstepping wavelet neural network control for indirect field-oriented induction motor drive. *IEEE Transactions on Neural Networks* **15** (2) (2004) 367–382.

[3] Hovakimyan, N., Lavretsky, N.E. and Cao. C Dynamic inversion of multi-input non affine systems via time scale separation. *Proceedings of the 2006 American Control Conference.* Minneapolis, Minnesota, USA (2006) 3594–3599.

[4] Young, A,, Cao. C, Hovakimyan, N. and Lavretsky, E. Control of a non affine double pendulum system via dynamic inversion and time scale separation. *Proceedings of the 2006 American Control Conference.* Minneapolis, Minnesota, USA (2006) 1820–1825.

[5] Yang, B. and Calise, A. Adaptive control of a class of non affine systems using neural networks. *Proceedings of the 44th IEEE conference on decision and control and the European control conference 2005,* Spain (2005) 2568–2573.

[6] Park, J.K., Huh, H.S. and Kim, S.H. Direct adaptive controller for non affine nonlinear systems using self structuring neural networks. *IEEE Transaction on neural networks* **16** (2) (2005) 414–421.

[7] Lee, C.H., Chin, J.C., Chang, H.H., Kuo, C.T. and Chang, H.H. Direct adaptive backstepping control for a class of mimo non affine systems using recurrent neural networks. *Proceedings of the IMECS* **1** (2009).

[8] Talmoudi, S., Abderrahim, K., Abdennour, R. Ben and Ksouri, M. Multimodal approach using neural networks for complex systems modelling and identification. *Nonlineqar Dynamics and Systems Theory* **8** (3) (2008) 299–316.

[9] Zhang, Q. and Benveniste, A. Wavelet networks. *IEEE Transactions on Neural Networks* **3** (6) (1992) 889–898.

[10] Zhang, J., Walter, G. G., Miao, Y. and Lee, W. Wavelet neural networks for function learning. *IEEE Transactions on Signal Processing* **43** (6) (1995) 1485–1497.

[11] Yoo, S. J., Park, J.B. and Choi, Y.H. Self predictive control of chaotic systems using self recurrent wavelet neural network. *International journal of control, automation and systems* **3** (1) (2005) 43–55.

[12] Yoo, S. J., Park, J.B. and Choi, Y.H. Direct adaptive control using self recurrent wavelet neural networks via adaptive learning rates for stable path tracking of mobile robots. *Proceedings of the 2005 American Control Conference.* Portland, OR, USA,(2005) 288–293.

[13] Astrom, K. J. and Wittenmark, B. *Adaptive control.* Addison Wesley, New York, 1995.

[14] Khalil, H.K. *Nonlinear systems.* Upper Saddle River, NJ Prentice Hall, 2002.

[15] Slotine, J.J.E. and Li, W. *Applied nonlinear control.* Prentice Hall, New Jersey, 1991.