Nonlinear Dynamics and Systems Theory, **11** (2) (2011) 113–123

Periodic Solutions of Singular Integral Equations

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Received: October 27, 2010; Revised: March 29, 2011

Abstract: We consider a scalar integral equation

$$x(t) = a(t) - \int_{-\infty}^{t} C(t,s)g(s,x(s))ds$$

in which $C(t,s)$ has a singularity at $t = s$. There are periodic assumptions on a, C, and g. First we prove a fixed point theorem of the Krasnoselskii–Schaefer type. We then construct a Liapunov functional which allows us to satisfy the conditions of the fixed point theorem and to prove that there is a periodic solution.

Keywords: *integral equations; fixed point theorems; periodic solutions; Liapunov functionals.*

Mathematics Subject Classification (2000): 45D05, 45D20, 45M15.

1 Introduction

We consider a scalar integral equation

$$x(t) = a(t) - \int_{-\infty}^{t} C(t,s)g(s,x(s))ds \tag{1}$$

for which there is a $T > 0$ so that

$$a(t+T) = a(t),\ g(t+T,x) = g(t,x),\ C(t+T,s+T) = C(t,s) \tag{2}$$

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