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Internal Multiple Models Control Based on Robust Clustering Algorithm

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Abstract: In this paper, Internal Multiple Model Control (IMMC) based on Robust Clustering Algorithm (RCA) is proposed. The IMMC requires, firstly, the definition of set a of local models each one valid in a given region. Different strategies exist in the literature dealing with the determination of the local models base. However, most of these strategies need a priori knowledge of the system. In order to overcome this difficulty, a RCA is proposed to find the optimum number of clusters. In the second step, the obtained data relative to each cluster will be used to build the local models base. Finally, the internal model control (IMC) structure will be developed using the models base where a linear controller will be constructed for every model. The efficiency of the IMMC based on RCA is demonstrated through an uncertain linear system and by the control of a neutralization of PH reaction in a Continuously Stirred Tank Reactor (CSTR).

Keywords: *IMMC; multiple models; RCA; PH neutralization system.*

Mathematics Subject Classification (2000): 93C42, 93B12, 92B20.

1 Introduction

In the case of linear plants, IMC have been extensively studied due to its robustness properties against disturbances and a model mismatch [21, 17, 7]. It uses the process model as the internal model to predict the process output. However, many industrial systems exhibit strong nonlinear behavior and they may be required to operate over a wide range of operating conditions. Additionally, there are situations where the nonlinear plants are extremely difficult to model and they exhibit high uncertainties [2, 3]. Under these

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conditions, the multi-model approach is an efficient and a powerful way to resolve problem of modeling and control of complex and non-linear processes [14, 15, 20, 10, 18, 16]. The past few years have shown an increase in the use of the multi-model representation combined with the IMC. The modeling concept includes a number of approaches such as: Takagi and Sugeno Fuzzy Inference Systems [14], local neural networks [1, 22]. However, these approaches remain so confronted with several difficulties such as the determination of the local models base. To resolve this problem, a RCA is proposed to determine the models base for complex systems. This approach is an unsupervised classification which does not require a priori knowledge about the system and uses a robust estimator to find the optimal number of clusters by repeatedly merging similar clusters [4]. The IMMC can be summarizing in three steps. The first step consists in dividing the systematic space in some subspaces using the RCA where a criterion is developed to find the optimal partition. In the second step, a local model is built for every subspace. Finally, the local models base will be combined with the IMC structure. We will show that IMMC has strong robustness under the conditions of modelling uncertainties. It can effectively compensate the modeling error of the plant by using this error as a feedback signal.

The remainder of this paper is organized as follows. In Section 2, we present the RCA. Section 3 presents a description of the IMMC and the validities computations. In Section 4, to check the ability of the proposed approaches, two examples have been considered. Finally, Section 5 provides the conclusion.

2 Robust Clustering Algorithm

The Robust Competitive Agglomeration (RCA) algorithm [5, 6] is a fuzzy partitional algorithm which does not require the number of clusters to be specified. Let $X = \{x_i/i \in \{1 \dots N\}\}$ be a set of N inputs vectors. Let $V = \{V_j/j \in \{1 \dots C\}\}$ represent prototypes of the clusters. The RCA algorithm minimizes the following objective function:

$$J_R(U,V) = \sum_{j=1}^{C}\sum_{i=1}^{N} (u_{ji})^2 \rho_j\left(d_{ji}^2\right) - \alpha \sum_{j=1}^{C}\left[\sum_{i=1}^{N} w_{ji}u_{ji}\right]^2. \tag{1}$$

In (1), d_{ji} stands for the distance from the input vector x_i to the center v_j and u_{ji} is the membership of x_i to cluster j. $\rho_j()$ is a robust loss function associated with cluster j, and $w_{ji} = w_j\left(d_{ji}^2\right) = \frac{\partial \rho_j\left(d_{ji}^2\right)}{\partial d_{ji}^2}$ represents the typicality of point x_i with respect to cluster j. The function $\rho_j\left(\right)$ corresponds to the loss function used in M-estimators of robust statistics and $w_j\left(\right)$ represents the weight function of an equivalent W-estimator [4]. By minimizing both terms in (1) simultaneously, the data set will be partitioned into the optimal number of clusters while clusters will be arranged in order to minimize the sum of intracluster distances [4].

Membership of cluster s can be written as [5]:

$$u_{st} = \frac{\frac{1}{\rho_s\left(d_{st}^2\right)}}{\sum\limits_{j=1}^{C} \frac{1}{\rho_j\left(d_{jt}\right)^2}} + \frac{\alpha}{\rho_s\left(d_{st}^2\right)}(N_s - \overline{N_t}) = u_{st}^{RR} + u_{st}^{Bias}, \tag{2}$$

where N_s represents the robust cardinality of the cluster s. It is defined by [5]:

$$N_s = \sum_{i=1}^{N} w_{si} u_{si}, \tag{3}$$

where $\overline{N_t}$ is the weighted average of cardinalities and it is defined by the following equation [5]:

$$\overline{N_t} = \frac{\sum\limits_{j=1}^{C} \frac{1}{\rho_j (d_{jt})^2} N_j}{\sum\limits_{j=1}^{C} \frac{1}{\rho_j (d_{jt})^2}}. \tag{4}$$

The first term in equation (2) is the membership term in the FCM algorithm using a robust distance. The second term leads to a reduction of cardinality of spurious clusters, which are discarded if their cardinality drops below a threshold [6]. So only good clusters are conserved. For clusters with cardinality higher than the average, the bias term is positive, thus appreciating the membership value. On the other hand, for low cardinality clusters, the bias term is negative, thus depreciating the membership value. It should be noted that when a feature point x_t is close to only one cluster s, and far from other clusters, we have:

$$N_s \approx N_t \Leftrightarrow u_{st}^{Bias} = 0. \tag{5}$$

In this case the membership value, u_{st} is independent of the cluster cardinalities, and is reduced to u_{st}^{RR}. In other words, if a point is close to only one cluster, it will have high membership value in this cluster and no competition is involved. On the other hand, if a point is close to many clusters, these clusters will compete for this point based on cardinality.

The parameter α should provide a balance between the two terms of (1), so α at iteration k is defined by [6]:

$$\alpha(k) = \eta_0 \exp\left(-\frac{k}{\tau}\right) \frac{\sum\limits_{j=1}^{C} \sum\limits_{i=1}^{N} \left(u_{ji}^{(k\text{-}1)}\right)^2 \rho_j\left(d_{ji}^2\right)}{\sum\limits_{j=1}^{C} \left[\sum\limits_{i=1}^{N} w_{ji}^{(k-1)} u_{ji}^{(k-1)}\right]^2}, \tag{6}$$

where η_0 is the initial value, and τ is the time constant. The exponential factor makes the second term preponderant in a first time to reduce the number of cluster, and then the first term dominates to seek the best partition of the data.

2.1 Weight function

The RCA technique proposed by Frigui and Krishnapuram [4, 5] tried to make the data partitioning robust by using the weight functions of a robust statistical law. The argument of the weight function consists of the squares of the distances. The weight function is chosen a monotonically nonincreasing function as defined below [4]:

$$w_j\left(d_{ji}^2\right) = \begin{cases} 1 - \frac{d_{ji}^4}{2T_j^2}, & if\; d_{ji}^2 \in [0, T_j], \\ \frac{\left[d_{ji}^2 - (T_j + cS_j)\right]^2}{2cS_j^2}, & if\; d_{ji}^2 \in \left]T_j, T_j + cS_j\right], \\ 0, & if\; d_{ji}^2 \succ T_j + cS_j, \end{cases} \tag{7}$$

where $T_j = MED_j\left(d_{ji}^2\right)$ and $S_j = MAD_j\left(d_{ji}^2\right)$, MED_j is the median of the residuals of the *j-th* cluster and MAD_j is the median of absolute deviations of the *j-th* cluster.

The loss function associated with this weight function can be derived by integrating (7). This yields:

$$\rho_j\left(d_{ji}^2\right) = \begin{cases} d_{ji}^2 - \frac{d_{ji}^6}{6T_j^2}, & if\ d_{ji}^2 \in [0, T_j], \\ \frac{\left[d_{ji}^2 - (T_j + cS_j)\right]^3}{6cS_j^2} + \frac{5T_j + cS_j}{6}, & if\ d_{ji}^2 \in]T_j, T_j + cS_j], \\ \frac{5T_j + cS_j}{6} + K_j, & if\ d_{ji}^2 \succ T_j + cS_j. \end{cases} \tag{8}$$

In (8) K_j is a constant used to make all $\rho_j\,()$ functions reach the same maximum value.

$$K_j = \max_{1 \le j \le C}\left\{\frac{5T_j + cS_j}{6}\right\} - \frac{5T_j + cS_j}{6}. \tag{9}$$

The constants K_j are added to prevent assigning all noise points to the most compact cluster. By forcing all functions to have the same maximum value, all noise points will have the same membership value in all clusters.

2.2 Algorithm outline

The RCA can be summarized by the following steps:
Step 1: Fix the maximum number of clusters, initialise the vector center, and set $k = 1$, w_{ji}=1 and c_0=12.
Step 2: Compute d_{ji} , estimate T_j and S_j.
Step 3: Update the weight w_j, $\alpha(k)$ and the partition matrix $U^{(k)}$.
Step 4: Compute the robust cardinality N_j , if $N_j \prec \varepsilon$ discard cluster *j*.
Step 5: Update the number of cluster C and $k = k + 1$.
Step 6: Update the tuning factor $c_k = \max(4, c_{k-1} - 1)$ and the center parameters.
Step 7 : Test of the convergence: if the center parameters stabilize then stop otherwise go to step 2.

The Mahanobis distance given by (10) has been used in this investigation. The update equations for the centers v_j and the covariance matrices are given by (11) and (12):

$$d^2\left(x_i, v_j\right) = |A_j|^{\frac{1}{n}}\left(x_i - v_j\right)^T A_j^{-1}\left(x_i - v_j\right), \tag{10}$$

$$v_j = \frac{\sum\limits_{i=1}^{N}\left(u_{ji}\right)^2 w_{ji} x_i}{\sum\limits_{i=1}^{N}\left(u_{ji}\right)^2 w_{ji}}, \tag{11}$$

$$A_j = \frac{\sum\limits_{i=1}^{N}\left(u_{ji}\right)^2\left(x_i - v_j\right)^T\left(x_i - v_j\right)}{\sum\limits_{i=1}^{N}\left(u_{ji}\right)^2 w_{ji}}. \tag{12}$$

2.3 Validity criteria

To assure an accurate modeling step, a robust competitive criterion D_c has been introduced. It is based on the comparison between the global cardinality and the average cardinality. The global cardinality of a partition is defined by:

$$C_G = \frac{1}{N}\sum_{i=1}^{N}\frac{\sum_{j=1}^{C}\frac{1}{\rho_j\left(d_{ji}^2\right)}N_j}{\sum_{j=1}^{C}\frac{1}{\rho_j\left(d_{ji}^2\right)}}. \tag{13}$$

The cardinality for one class represents the average distribution of the points around the center. Then the average cardinality is computed from the cardinality of all the classes in order to obtain a value which translates the distribution of the points around the center of the classes. The average cardinality of the created clusters is defined by:

$$C_V = \frac{1}{C}\sum_{j=1}^{C}\sum_{i=1}^{N} w_{ji}u_{ji} \tag{14}$$

The average and the global cardinality are equivalent on condition that the partitioning is optimal. That is to say when the obtained clusters correspond to the real classes, the ratio between average and global cardinality tends towards one.

The robust criterion validity D_c which reflects the state of the partition is given by:

$$D_c = \left|1 - \frac{C_G}{C_V}\right|. \tag{15}$$

When the optimal partition is attained the ratio in (15) tends to *1* and so the criterion D_c is minimal. So, the optimal partition is obtained for the minimum value of D_c.

3 The Principle of IMMC Based on Classifier

In IMC, a plant model is placed in parallel with the real plant [17]. The difference between the plant and the model outputs is used for feedback purposes. The feedback signal is an estimate of the plant disturbances or the model mismatch. For linear plant, IMC have been shown good robustness properties against disturbances and model mismatches [19]. In this paper, the linear IMC strategy will be investigated to control uncertain systems using multiple models. According to the characteristics of the controlled plant, the design principle of IMMC based on RCA is as follows, firstly, divide the data into some subspaces using the RCA. The second step is a structural and parametric estimations step in order to determine the local base models. In fact, a partional linear model is built to every subspace. In the third step, the IMC structure will be combined with the local models base where a linear controller will be constructed for every local model.

The filter F in the IMC structure is employed to introduce robustness into the controller to deal with plant uncertainty [11].

3.1 Development of local linear models

The core idea is to represent the uncertain nonlinear dynamic system by a set of locally valid sub-linear models across the operating range. Each model is developed around

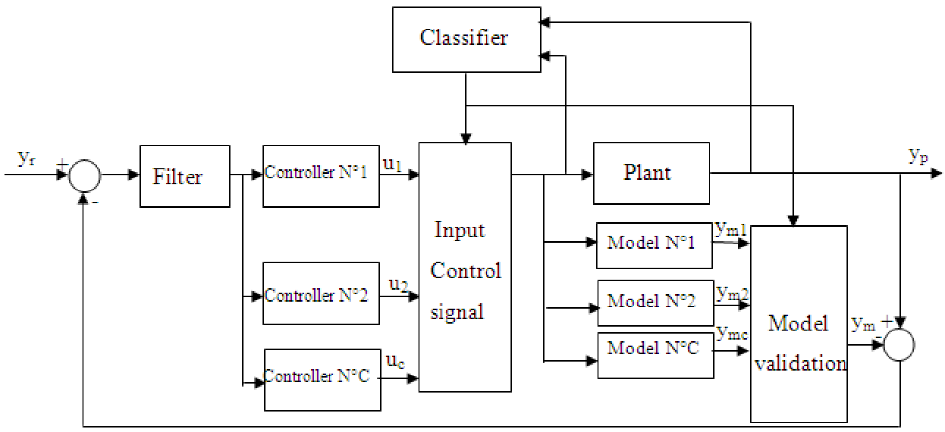

Figure 1: Structure of IMMC.

an operation range. A structural and a parametric identification must be carried out to elaborate the related local model. The established models are constructed using the ARX structure given by the following relation:

$$y(k) = -\sum_{i=1}^{n} a_i y(k-i) + \sum_{j=1}^{m} b_j u(k-j), \tag{16}$$

where u is the input to the unknown system, y is the output system, a_i and b_j are the parameters of the ARX model. The parametric identification uses the Recursive Least Square (RLS) method and exploits the observation-vectors related to every cluster.

3.2 Development of local controllers

For minimum-phase processes (stable with no time delay or zeros outside the unit circle), IMC can produce perfect control based on a controller designed as the inverse of the process. When dealing with a non minimum phase process, this procedure cannot be used directly since the transfer function of plant $\hat{G}$ is not invertible. One approach to handle a noninvertible process model is to apply the following factorization

$$\hat{G}(z) = \hat{G}^{-} \; \hat{G}^{+}, \tag{17}$$

where $\hat{G}^{+}$ contains all the zeros outside the unit circle and all the time delays and $\hat{G}^{-}$ is then inverted for controller design [9].

3.3 Control strategies

In IMMC, two control strategies are considered. The first one is based on the switching between different models. This method consists in choosing the nearest model to the process which leads to the least modelling error. The appropriate controller is then obtained from the validated inverse model. In the second strategy, the internal model

and the controller outputs are obtained by fusion of generated models and controllers pondered by a validity criterion. The global model and control outputs are presented by the following equations.
In the first strategy:

$$y_m(k) = y_{mj}(k), \quad where \quad d_{jk} = \min_{1\leq t\leq C}(d_{tk}), \tag{18}$$

$$u_c(k) = u_j(k), \quad where \quad d_{jk} = \min_{1\leq t\leq C}(d_{tk}). \tag{19}$$

In the second strategy:

$$y_m(k) = \sum_{j=1}^{C} r_j(k)\; y_{mj}(k), \tag{20}$$

$$u_c(k) = \sum_{j=1}^{C} r_j(k)\; u_j(k), \tag{21}$$

where C is the number of models in the library. y_{mj} and u_j are respectively the outputs of model M_j and its corresponding controller. r_j is the validity criterion.

Several validities computation methods were proposed in the literature [8, 13, 12]. All these methods are based on measuring the distance between the current state of the process and the model M_j. The proposed method of validities computation is inspired from the fuzzy version where the cluster's parameters obtained from the RCA are exploited. It evaluates the contribution of the model to describe the system behaviors in its full range

$$r_j(k) = \frac{1 - \frac{d_{ji}}{\sum_{j=1}^{C} d_{ji}}}{C-1}. \tag{22}$$

4 Simulation Results

In order to evaluate the performances of the presented algorithms, two examples will be considered. The first one concerns the control of an uncertain linear system. The second example treats the control of a chemical process which is a PH neutralization process.

4.1 Uncertain linear system

Let us consider a linear system with uncertain parameters described by [20]:

$$y^p(k) = -a_1(k)y^p(k-1) - a_2(k)y^p(k-2) + b_1(k)u(k-1) + b_2(k)u(k-2), \tag{23}$$

$$a_1(k) = -0.8 + 0.08 \sin\left(\frac{2\pi k}{200}\right), \tag{24}$$

$$a_2(k) = 0.1 + 0.01 \sin\left(\frac{2\pi k}{200}\right), \tag{25}$$

$$b_1(k) = 0.5 + 0.04 \sin\left(\frac{2\pi k}{200}\right), \tag{26}$$

$$b_2(k) = 0.2 + 0.02 \sin\left(\frac{2\pi k}{200}\right). \tag{27}$$

The system is excited by a random sequence $[0, 3]$ to generate the necessary data base. 2000 samples are used where $[y^p(k),\, y^p(k-1), u(k-1), u(k-2)]$ is the vector to be clustered. The minimization of the criterion D_c allows to find the optimal partition corresponding to the number of clusters. Table 1 illustrates the results obtained for this example where the minimum is detected for two clusters.

Number of cluster	2	3	4	5	6
D_c	0.08	0.401	0.46	0.586	0.667

Table 1: D_c criterion of the first example.

The clustered data based on D_c criterion are presented in Figure 2.

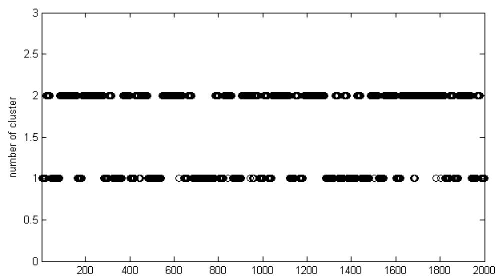

Figure 2: Clustered data based on RCA (example 1).

The following stage is to estimate the parameters of each local model. A second order ARX model has been used as the model structure. The parameters of each local model are given in Table 2.

Models	M_1	M_2
a_1	1.2973	1.6661
a_2	-0.4457	-0.7151
b_1	0.5203	0.5064
b_2	-0.1706	-0.396

Table 2: Model Parameters of the first example.

In order to compare performances of the control strategies, a criterion will be defined and given by the following equation:

$$E = \frac{1}{N} \sum_{k=1}^{N} [y^r(k) - y^p(k)]^2. \tag{28}$$

The responses of IMMC using the first and the second control strategies are given by the following figures.

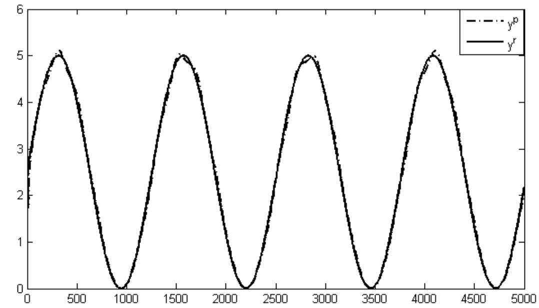

Figure 3: IMMC output evolution using the first strategy for set point tracking.

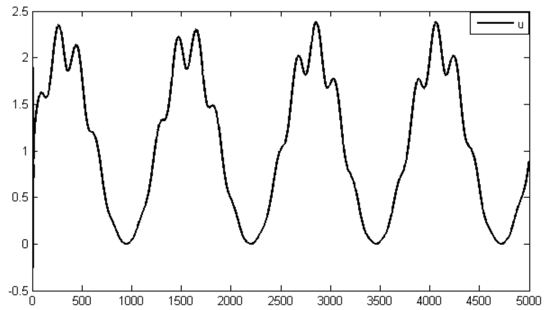

Figure 4: IMMC control input evolution using the first strategy.

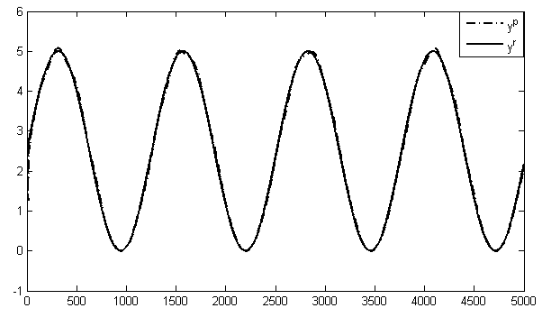

Figure 5: IMMC output evolution using the second strategy for set point tracking.

In the first strategy control $E = 0.0658$. In the second strategy control $E = 0.0522$. These figures show that the plant output y^p follows the desired output y^r of the uncertain process. The obtained performances show that second approach is slightly more accurate.

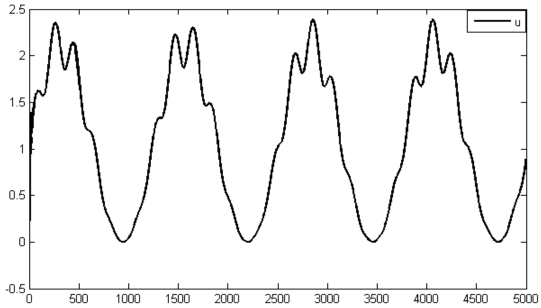

Figure 6: IMMC control input evolution using the second strategy.

4.2 Uncertain nonlinear system: PH neutralization system

In order to show the performance of the IMMC, it will be applied to the case of the PH neutralization system. It is a well-known benchmark problem and it has two input streams: sodium hydroxide and acetic acid. For collection of the data, a sampling time of 12 second has been used [1].

Parameters	Description	Nominal Value
v	Volume of the tank	1000 [l]
q_1	Flow rate of acetic acid	81 [l/min]
C2	Inlet concentration of NaOH	0.05 [mol/l]
C1	Inlet concentration of acetic acid	0.32 [mol/l]
C_A	Initial concentration of sodium in the CSTR	0.0432 [mol/l]
C_B	Initial concentration of acetate in the CSTR	0.0432 [mol/l]
K_a	Acid equilibrium constant	1.753 10-5
K_b	Inlet concentration of acetic acid	10-14

Table 3: Parameters description.

Number of cluster	2	3	4	5	6
D_c	0.5899	0.0303	0.2088	0.1307	0.156

Table 4: D_c criterion for the second example.

The process model consists of two nonlinear ordinary differential equations and a nonlinear output equation for the PH.

$$\begin{cases} v\frac{dC_A}{dt} = q_1C_1 - (q_1+q_2)C_A, \\ v\frac{dC_B}{dt} = q_2C_2 - (q_1+q_2)C_B, \\ 10^{3PH} + 10^{2PH}\,(K_a + C_B) + 10^{PH}(C_BK_a - C_AK_a - K_w) - K_a + K_w. \end{cases} \tag{29}$$

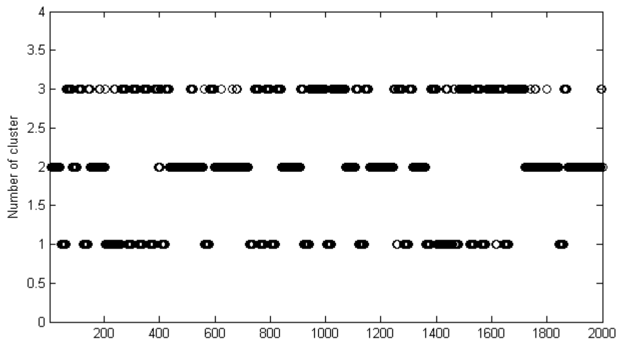

Figure 7: Clustered data based on RCA (example 2).

Models	M_1	M_2	M_3
a_1	1.15	1.16	1.131
a_2	-0.2379	-0.2148	-0.1797
b_1	0.01025	0.04474	0.009975
b_2	0.001234	- 0.01256	- 0.004141

Table 5: Model Parameters of the second example.

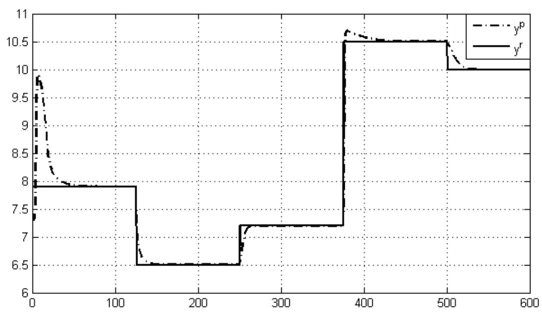

Figure 8: IMMC output evolution for PH tracking using the first strategy.

The process output is the PH and the input is the sodium flowrate q_2. The parameters used in the simulations are described and given in Table 3. The system is excited by a control input q_2 of random amplitude in the range $[512; 525]$ with duration of 20 sampling periods; the total length of the sequence is 2000. Three clusters are identified. The number of cluster was determined using the Table 4 where the minimum is detected for three clusters. The clustering results of the PH neutralization system are presented in the Figures 7.

The model basis considered consists of three local models. They are described by

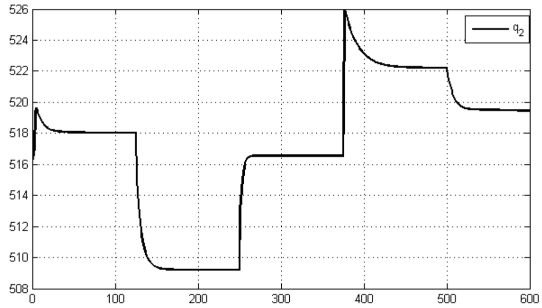

Figure 9: IMMC control input evolution using the first strategy.

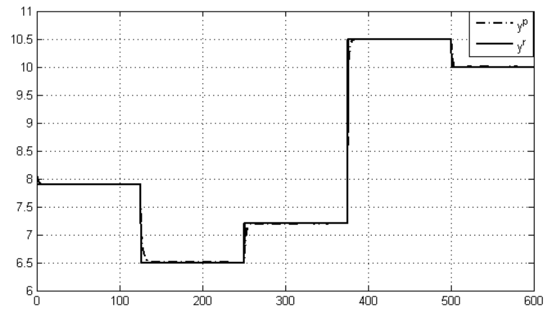

Figure 10: IMMC output evolution for PH tracking using the second strategy.

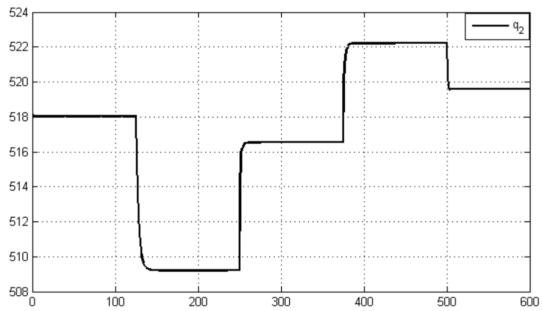

Figure 11: IMMC control input evolution using the second strategy.

discrete transfer functions having the same structure where the parameters of the process are given by the Table 5. The PH responses of the system using different strategies are shown in Figures 8 and 10. In the first strategy control $E = 0.0753$. In the second strategy control $E = 0.0079$.

Using the first strategy, Figure 8 shows that there is clearly a poor performance. However, when models are pondered more satisfactory tracking behavior (especially in control action) is detected. Indeed, the obtained relative errors confirm the robustness of the second strategy. The difference from the previous simulations can be attributed to the model plant mismatch.

5 Conclusion

Considering the complexity to control uncertain plants, IMMC based on RCA is proposed in view of all the advantage of multiple model and internal model control. The IMMC can be applied in three steps. The primary step consists in determining the suitable number of local base models using the RCA. The second step is parametric estimations step in order to determine the local base models. In fact, a partional linear model is created to every subspace. In the third step, the IMC structure will be combined with the local base models where a controller will be constructed for every model. The application of this approach is carried out on two simulation examples of uncertain systems. The simulation results show that IMMC is more robust when the models and controllers are weight on global model and controller.

References

[1] Bhat, N.V. and McAvoy, T.J. Determining model structure for neural models by network stripping. *Computers and Chemical Engineering* **16** (1992) 271–281.

[2] Boukezzoula, R., Galichet, S. and Foulloy L. Nonlinear Internal Model Control: Application of Inverse Model Based Fuzzy Control. *IEEE Transactions on Fuzzy Systems* **11** (6) (2003) 814–829.

[3] Brown, M.D., Lightbody, G. and Irwin G.W. Nonlinear internal model control using local model networks. *IEEE Proc. Control Theory Appl.* **144** (6) (1997) 505–514.

[4] Frigui,H. and Krishnapuram, R. A Robust Competitive Clustering Algorithm With Applications in Computer Vision. *IEEE Trans. Pattern Anal. Mach. Intell* **21** (5) (1999) 450–465.

[5] Frigui, H. and Krishnapuram, R. A robust algorithm for automatic extraction of an unknown number of clusters from noisy data. *Pattern Recognition Letters* **17** (1996) 1223–1232.

[6] Frigui, H. and Krishnapuram, R. Clustering by competitive agglomeration. *Pattern Recognition* **30** (7) (1997) 1109–1119.

[7] Gala, O., Romagnoli, J.A., Palazoglu, A. and Arkun Y. Gap Metric Concept and Implications for Multilinear Model-Based Controller Design. *Ind. Eng. Chem. Res.* **42** (2003) 2189–2197.

[8] Gasso, K. *Identification des systmes dynamiques non linaires: Approche multimodle*. Thesis, Institut National Polytechnique de Lorraine, France, 2000.

[9] Garcia, C.E. and Moran, M. Internal Model Control. I. A Unifying Review and Some Neu Result.*Ind. Eng. Chem. Process Des. Dev.* **21** (1982) 308–323.

[10] Gawthrop, P.J. Continuous-time local state local model networks. *Proceeding of IEEE Conference on Systems, Man and Cybernetics* (1995) 852–857.

[11] Hunt, K.J. and Sbarbaro, D. Adaptive filtering and neural networks for realisation of internal model control. *Intelligent Systems Engineering* **2** (2) (1993) 67–76.

[12] Johansen, T.A. and Foss, B.A. Constructing NARMAX models using ARMAX models. *Int. J. Control* **58** (1993) 1125–1153.

[13] Johansen, T.A. and Foss, B.A. Multiple model approaches to modelling and control. *Int. J. Control* **72** (1999) 575.

[14] Jun, L., Wan-li, W. and Xiu-hua, D. Multiple Model Internal Model Control Based on Fuzzy Membership Function. *Proceedings of the IEEE International Conference on Automation and Logistics Qingdao*, 2008.

[15] Lahmari, K. *Contribution la commande multi-modle des processus complexes.* Thesis, UST de Lille, 1999.

[16] Mezghani, S., Elkamel, A. and Borne, P. Multi-model control of discrete systems with uncertainties. *The Electronic International Journal of Advance Modilling and Optimization* **32** (2001) 7–17.

[17] Morari, M. and Zafiriou, E. *Robust Process Control.* Englewood Cliffs, NJ: Prentice-Hall, 1989.

[18] Narendra, K.S. and Balakrishnan. Adaptative Control Using Multiple Models. *IEEE Transactions on Automatic Control* **42** (1997) 171–187.

[19] Rivals, I. and Personnaz, L. Nonlinear internal model control using neural networks: Application to processes with delay and design issues. *IEEE Transactions on Neural Networks* **11** (2000) 80–90.

[20] Talmoudi, S., Abderrahim, k., Ben Abdennour, R; and Ksouri, M. Multi-model Approach using Neural Networks for Complex Systems Modeling and Identification. *Nonlinear Dynamics and Systems Theory* **8** (3) (2008) 299–316.

[21] Wang, Q.G., Bi, Q. and Zhang, Y. Partial Internal Model Control. *IEEE Transactions on Industrial Electronics* **48** (5) (2001).

[22] Zhao, Z., Liu, Z., Wen, X. and Zhang, J. *A new multi-model internal model control scheme based on neural network.* Proceedings of the 7th World Congress on Intelligent Control and Automation, Chongqing, China, 2008.