Nonlinear Dynamics and Systems Theory, **10** (4) (2010) 387–396

Synchronization of Chaotic Systems by the Generalized Hamiltonian Systems Approach◇

S.H. Fu * and L.J. Pei

Department of Mathematics
Zhengzhou University, Zhengzhou, Henan 450001, PR China

Received: September 6, 2009; Revised: October 14, 2010

Abstract: In this paper, the generalized Hamiltonian system approach was applied to the synchronization of chaotic systems. The synchronization is between the transmitter and the receiver dynamics. The synchronization of several chaotic systems is studied by the method, respectively. The numerical results are in very good agreement with the theoretical analysis.

Keywords: *chaotic system; chaotic synchronization; generalized Hamiltonian system.*

Mathematics Subject Classification (2000): 37N35, 65P20, 68P25, 70K99, 93D20, 94A99.

1 Introduction

In the 17th century, the analysis of synchronization phenomena in the evolution of dynamical systems was a subject of active investigation [1]. Recently, the search for synchronization has moved to chaotic systems. Synchronization of chaos refers to a process wherein two (or many) chaotic systems adjust a given property of their motion to a common behavior due to a coupling or to a forcing.

The first thing to be highlighted is that there is a great difference in the process leading to synchronized states, depending upon the particular coupling configuration [1]. Namely, one should distinguish two main cases: unidirectional coupling and bidirectional coupling. In the former case, one subsystem evolves freely and drives the evolution of the other; in the latter case, both subsystems are coupled with each other.

◇ This work is supported by National Natural Science Foundation of China (No. 10702065).
* Corresponding author: mailto:fshtp@163.com

In the context of coupled chaotic elements, many different synchronization states have been studied in the past 10 years: namely complete or identical synchronization [2]-[4], phase [5, 6] and lag synchronization [7], generalized synchronization [8, 9], intermittent lag synchronization [7, 10], imperfect phase synchronization [11], and almost synchronization [12]. Complete synchronization was the first discovered and is the simplest form of synchronization in chaotic systems. It consists in a perfect hooking of the chaotic trajectories of two systems which is achieved by means of a coupling signal, in such a way that they remain in step with each other in the course of time.

The phenomena of chaotic synchronization exists widely in laboratory experiments and natural systems [13]-[22]. The natural continuation of the pioneering works was to investigate synchronization phenomena in spatially extended or infinite dimensional systems [13]-[16], to test synchronization in experiments or natural systems [17]-[22]. The synchronization has also been applied to encoding or masking where the chaotic system is called the "transmitter". Correspondingly for the decoding or unmasking, the second chaotic system is called the "receiver". The synchronization between the "transmitter" and the "receiver" means that, under the assumption of no masked signal transmission, the receiver state trajectory asymptotically tracks that of the transmitter. In [23], the authors have studied the synchronization of two chaotic systems by the generalized Hamiltonian system and observer approach. Furthermore, the method is extended to the time-delay Chua's oscillator [24].

The objective of this paper is to apply the generalized Hamiltonian system and observer approach developed in [23] to the complete synchronization of two identical chaotic systems coupled unidirectionally. The organization of the paper is as follows: In Section 2, we obtain the synchronization of chaotic systems by the generalized Hamiltonian system and observer approach. In Section 3, we present several chaotic systems and study their synchronization by this method, respectively. In Section 4, the conclusion is given.

2 The Synchronization of Chaotic Systems

A smooth system is given as follows:

$$\dot{x} = f(x,t), \quad x = (x_1, x_2 \ldots x_n)^T \in R^n, \tag{1}$$

where $f \in R^n$ is smooth.

Equation (1) may be written in the generalized Hamiltonian system:

$$\dot{x} = J_1(x)\frac{\partial H}{\partial x} + S(x)\frac{\partial H}{\partial x} + F_1(x,t), \tag{2}$$

where $H(x)$ denotes a smooth energy function and is globally positive definite in R^n, and the column gradient vector $\frac{\partial H}{\partial x}$ of $H(x)$ is assumed to exist everywhere; if the form of quadratic energy function is $H = \frac{1}{2}x^T Mx$ (M is a constant symmetric positive definite matrix), $\frac{\partial H}{\partial x} = Mx$. $J_1(x) + J_1^T(x) = \theta$, $S(x) = S^T(x)$. The vector field $J(x)\frac{\partial H}{\partial x}$ exhibits the conservative part of the system and it is also referred to as the workless part; and $S(x)$ depicts the working part of the system. $F_1(x,t)$ is a locally destabilizing vector field. According to the form of $H(x)$ and the different expression of $J_1(x)$, $S(x)$, $F_1(x,t)$, the form of the Generalized Hamiltonian system (2) is not unique.

In the context of observer design, we consider a special class of Generalized Hamiltonian system with liner output map y:

$$\begin{cases} \dot{x} = J(y)\frac{\partial H}{\partial x} + (I+S)\frac{\partial H}{\partial x} + F(y,t), \\ y = C\frac{\partial H}{\partial x}, \end{cases} \tag{3}$$

where $J(x)+J^T(x)=\theta$, I is a constant skew symmetric matrix, S is a constant symmetric matrix, and $F(x,t)$ is a locally destabilizing vector field. The vector variable y is referred to as the system output, and the matrix C is a constant matrix. Equation (3) is called the transmitter.

Let ξ and μ be the estimates of the state vector x and output y, respectively; and $\frac{\partial H}{\partial \xi} = M\xi$ is naturally the gradient of the Hamiltonian energy function $H(\xi)$). A dynamic nonlinear state observer for (3) is obtained as:

$$\begin{cases} \dot{\xi} = J(y)\frac{\partial H}{\partial \xi} + (I+S)\frac{\partial H}{\partial \xi} + F(y,t) + K(y-\eta), \\ \eta = C\frac{\partial H}{\partial \xi}, \end{cases} \tag{4}$$

where K is a constant matrix, known as the observer gain. Equation(4) is called the receiver.

In this paper, we study mainly the synchronization of the transmitter (3) and the receiver (4). Practically, it is the complete synchronization of two identical chaotic systems coupled unidirectionally.

Let $e(t) = x(t) - \xi(t)$, $e_y = y - \eta$, then the state estimation error [23] are governed by

$$\begin{cases} \dot{e} = \left(J(y) + I - \frac{1}{2}(KC - C^TK^T)\right)\frac{\partial H}{\partial e} + \left(S - \frac{1}{2}(KC + C^TK^T)\right)\frac{\partial H}{\partial e}, \\ e_y = C\frac{\partial H}{\partial e} \qquad e_y \in R^m \quad , \end{cases} \tag{5}$$

where $\frac{\partial H}{\partial e} = \frac{\partial H}{\partial x} - \frac{\partial H}{\partial \xi} = M(x-\xi) = Me$.

In [1], the authors point out that the transmitter (3) synchronizes with the receiver (4), if $\lim_{t\to\infty} \|x(t) - \xi(t)\| = 0$ no matter which initial conditions $x(0)$ and $\xi(0)$ have. The state estimation error $e(t) = x(t) - \xi(t)$represents the synchronization error. So we will study the system (5) for the synchronization. In the following, two theorems about (5) give the condition under which their synchronization happens. Let $W = I + S$.

Theorem 2.1 [23] *The state $x(t)$of the nonlinear system* (3) *can be globally exponentially asymptotically estimated by the state $\xi$of the nonlinear observer* (4)*, if the pair of matrices* (C, W)*or the pair* (C, S)*, is either observable or, at least, detectable.*

An observability condition on either of the pairs (C, W) or (C, S), is clearly a sufficient but not necessary condition for asymptotic state reconstruction. A necessary and sufficient condition for global asymptotic stability to zero of the estimation error is given by the following theorem.

Theorem 2.2 [23] *The state $x(t)$of the nonlinear system* (3) *can be globally exponentially asymptotically estimated by the state $\xi$of the nonlinear observer* (4)*, if and only if there exists a constant matrix K such that the symmetric matrix*

$$[W - KC] + [W - KC]^T = [S - KC] + [S - KC]^T = 2\left[S - \frac{1}{2}(KC + C^TK^T)\right] \tag{6}$$

is negative definite.

3 Numerical Application

3.1 The forced Brusselator

The equation of this system is given as follows [25]:

$$\begin{cases} \dot{x}_1 = A - (B+1)x_1 + x_1^2 x_2 + a\cos(\omega t), \\ \dot{x}_2 = Bx_1 - x_1^2 x_2. \end{cases} \tag{7}$$

After taking as a Hamiltonian energy function the scalar function $H(x) = \frac{1}{2}(x_1^2 + x_2^2)$, we obtain:

$$J(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, I = \begin{bmatrix} 0 & \frac{-B}{2} \\ \frac{B}{2} & 0 \end{bmatrix}, S = \begin{bmatrix} -(B+1) & \frac{B}{2} \\ \frac{B}{2} & 0 \end{bmatrix}, F(x) = \begin{bmatrix} A + x_1^2 x_2 + a\cos(\omega t) \\ -x_1^2 x_2 \end{bmatrix}.$$

We choose $y = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$, then $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, thus $K = \begin{bmatrix} K_1 & K_3 \\ K_2 & K_4 \end{bmatrix}$. The system is in generalized Hamiltonian canonical form:

$$\dot{x} = J(x)\frac{\partial H}{\partial x} + (I+S)\frac{\partial H}{\partial x} + F(x,t), \tag{8}$$

and the receiver is

$$\dot{\xi} = J(x)\frac{\partial H}{\partial \xi} + (I+S)\frac{\partial H}{\partial \xi} + F(x,t) + K(x-\xi), \tag{9}$$

The synchronization error, corresponding to this receiver, is

$$\begin{aligned} \dot{e} &= \left(J(x) + I - \tfrac{1}{2}(KC - C^T K^T)\right)\tfrac{\partial H}{\partial e} + \left(S - \tfrac{1}{2}(KC + C^T K^T)\right)\tfrac{\partial H}{\partial e} \\ &= \begin{bmatrix} 0 & \frac{-B-K_3+K_2}{2} \\ -\frac{-B-K_3+K_2}{2} & 0 \end{bmatrix}\tfrac{\partial H}{\partial e} + \begin{bmatrix} -(B+1) - K_1 & \frac{B-(K_2+K_3)}{2} \\ \frac{B-(K_2+K_3)}{2} & 0 \end{bmatrix}\tfrac{\partial H}{\partial e}. \end{aligned} \tag{10}$$

The pair (C, S) is observable, and hence detectable. We could prescribe K_1,K_2,K_3 andK_4, in order to ensure asymptotic stability of equation(8) and equation(9) to zero of the synchronization error. By applying Theorem 2.2, we obtain

$$2\begin{bmatrix} -(B+1) - K_1 & \frac{B-(K_2+K_3)}{2} \\ \frac{B-(K_2+K_3)}{2} & 0 \end{bmatrix}$$

is negative definite, i.e. $K_1 > -(B+1); 4K_4[(B+1) + K_1] > (B - K_2 - K_3)^2$.

In Figure 1, the synchronization of two chaotic systems (8) and (9) is presented. The parameters were taken as: $A = 0.4, B = 1.2, \omega = 0.8, a = 0.05, K_1 = 0.8, K_2 = 0.2, K_3 = 1, K_4 = 1$.

3.2 The forced pendulum

The equation of this system is given as follows [26]:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = -ax_2 - b\sin x_1 + \rho\cos(\omega t). \end{cases} \tag{11}$$

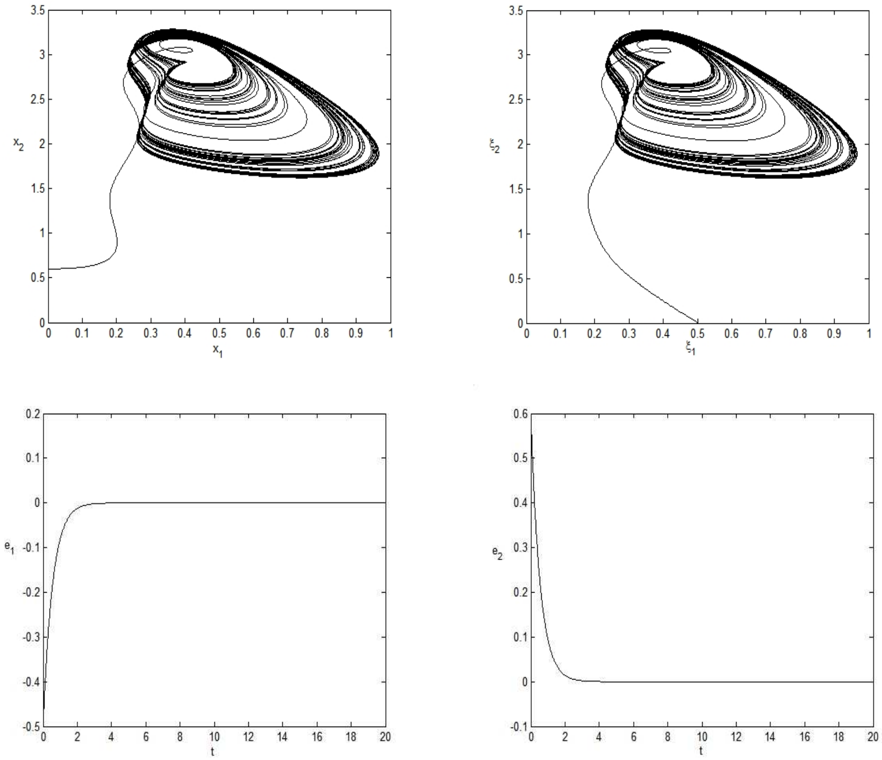

Figure 1: The synchronization of the forced Brusselator systems (8) and (9).

After taking as a Hamiltonian energy function the scalar function $H(x) = \frac{1}{2}(x_1^2 + x_2^2)$, we obtain:

$$J(x) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix},\ I = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{-1}{2} & 0 \end{bmatrix},\ S = \begin{bmatrix} 0 & \frac{1}{2} \\ \frac{1}{2} & -a \end{bmatrix},\ F(x) = \begin{bmatrix} 0 \\ -b\sin x_1 + \rho\cos(\omega t) \end{bmatrix}.$$

We choose $y = [x_1]$, then $C = \begin{bmatrix} 1 & 0 \end{bmatrix}$, thus $K = \begin{bmatrix} K_1 \\ K_2 \end{bmatrix}$. The system is in generalized Hamiltonian canonical form:

$$\dot{x} = J(x)\frac{\partial H}{\partial x} + (I + S)\frac{\partial H}{\partial x} + F(x,t), \tag{12}$$

and the receiver is

$$\dot{\xi} = J(x)\frac{\partial H}{\partial \xi} + (I + S)\frac{\partial H}{\partial \xi} + F(x,t) + K(x_1 - \xi_1). \tag{13}$$

The synchronization error, corresponding to this receiver, is

$$\begin{aligned} \dot{e} &= \left(J(x) + I - \tfrac{1}{2}(KC - C^T K^T)\right)\tfrac{\partial H}{\partial e} + \left(S - \tfrac{1}{2}(KC + C^T K^T)\right)\tfrac{\partial H}{\partial e} \\ &= \begin{bmatrix} 0 & \frac{1+K_2}{2} \\ -\frac{1+K_2}{2} & 0 \end{bmatrix}\tfrac{\partial H}{\partial e} + \begin{bmatrix} -K_1 & \frac{1-K_2}{2} \\ \frac{1-K_2}{2} & -a \end{bmatrix}\tfrac{\partial H}{\partial e}. \end{aligned} \tag{14}$$

The pair (C, S) is observable, and hence detectable. We could prescribe K_1 and K_2, in order to ensure asymptotic stability of equation (12) and equation (13) to zero of the synchronization error. By applying Theorem 2.2, we obtain

$$2\begin{bmatrix} -K_1 & \frac{1-K_2}{2} \\ \frac{1-K_2}{2} & -a \end{bmatrix}$$

is negative definite, i.e. $K_1 > 0; 4aK_1 - (1-K_2)^2 > 0$.

In Figure 2, the synchronization of two chaotic systems (12) and (13) is presented. The parameters were taken as: $a = 0.2, b = 1, \rho = 1.5, \omega = 0.4, K_1 = 2, K_2 = 2$.

3.3 The 3D model

The model is described by the equation as follows [27]:

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = -x_2 - 1.2x_3 - \mu x_1 + x_1^2 - 1.425x_2^2 + 0.2x_1x_3 - 0.01x_1^2x_3. \end{cases} \tag{15}$$

After taking as a Hamiltonian energy function the scalar function $H(x) = \frac{1}{2}(x_1^2+x_2^2+x_3^2)$, we obtain:

$$J(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, I = \begin{bmatrix} 0 & \frac{1}{2} & \frac{\mu}{2} \\ -\frac{1}{2} & 0 & 1 \\ \frac{-\mu}{2} & -1 & 0 \end{bmatrix}, S = \begin{bmatrix} 0 & \frac{1}{2} & -\frac{\mu}{2} \\ \frac{1}{2} & 0 & 0 \\ \frac{-\mu}{2} & 0 & -1.2 \end{bmatrix},$$

$$F(x) = \begin{bmatrix} 0 \\ 0 \\ x_1^2 - 1.425x_2^2 + 0.2x_1x_3 - 0.01x_1^2x_3 \end{bmatrix}.$$

We choose $y = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then $C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$, thus $K = \begin{bmatrix} K_1 & K_4 & K_7 \\ K_2 & K_5 & K_8 \\ K_3 & K_6 & K_9 \end{bmatrix}$.

The system is in generalized Hamiltonian canonical form:

$$\dot{x} = J(x)\frac{\partial H}{\partial x} + (I+S)\frac{\partial H}{\partial x} + F(x,t), \tag{16}$$

and the receiver is

$$\dot{\xi} = J(x)\frac{\partial H}{\partial \xi} + (I+S)\frac{\partial H}{\partial \xi} + F(x,t) + K(x-\xi). \tag{17}$$

The synchronization error, corresponding to this receiver, is

$$\begin{aligned} \dot{e} &= \left(J(x) + I - \tfrac{1}{2}(KC - C^TK^T)\right)\tfrac{\partial H}{\partial e} + \left(S - \tfrac{1}{2}(KC + C^TK^T)\right)\tfrac{\partial H}{\partial e} \\ &= \begin{bmatrix} 0 & \frac{1+K_2-K_4}{2} & \frac{\mu+K_3-K_7}{2} \\ -\frac{1+K_2-K_4}{2} & 0 & 1-\frac{K_8-K_6}{2} \\ -\frac{\mu+K_3-K_7}{2} & -1+\frac{K_8-K_6}{2} & 0 \end{bmatrix}\tfrac{\partial H}{\partial e} \\ &+ \begin{bmatrix} -K_1 & \frac{1-K_2-K_4}{2} & -\frac{\mu+K_3+K_7}{2} \\ \frac{1-K_2-K_4}{2} & -K_5 & \frac{-K_8-K_6}{2} \\ -\frac{\mu+K_3+K_7}{2} & \frac{-K_8-K_6}{2} & -1.2-K_9 \end{bmatrix}\tfrac{\partial H}{\partial e}. \end{aligned} \tag{18}$$

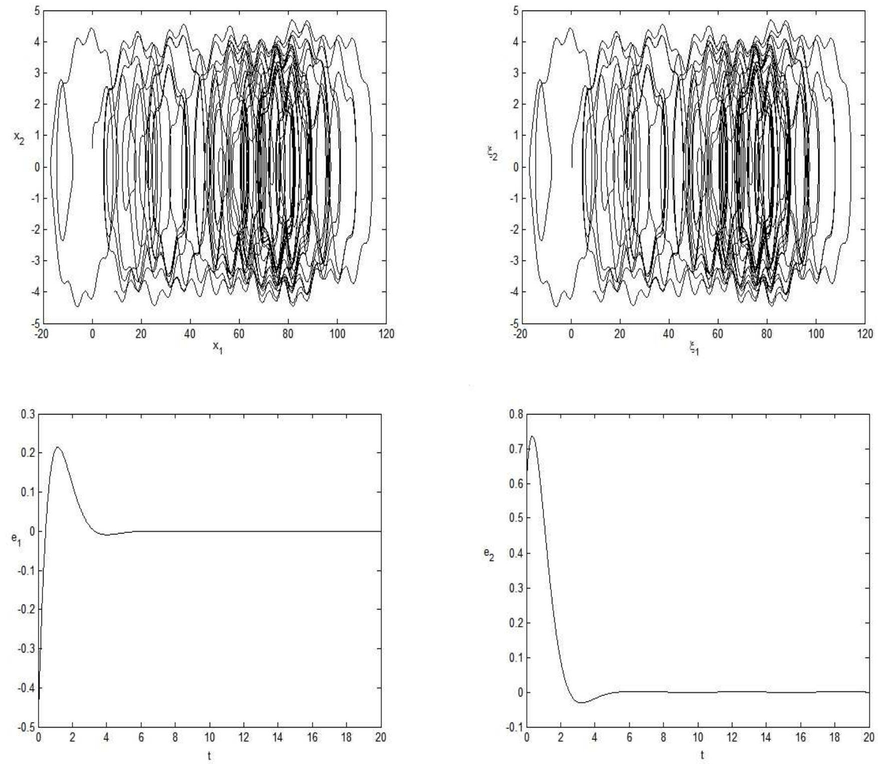

Figure 2: The synchronization of the forced pendulum systems (12) and (13).

The pair (C, S) is observable, and hence detectable. We could prescribe K_1, K_2, K_3, K_4, K_5, K_6, K_7, K_8, K_9 in order to ensure asymptotic stability of equation (16) and equation (17) to zero of the synchronization error. By applying Theorem 2.2, we obtain

$$2\begin{bmatrix} -K_1 & \frac{1-K_2-K_4}{2} & -\frac{\mu+K_3+K_7}{2} \\ \frac{1-K_2-K_4}{2} & -K_5 & \frac{-K_8-K_6}{2} \\ -\frac{\mu+K_3+K_7}{2} & \frac{-K_8-K_6}{2} & -1.2-K_9 \end{bmatrix}$$

is negative definite, i.e.

$$\begin{aligned} &K_1 > 0, \\ &4K_1K_5 > (1-K_2-K_4)^2, \\ &(1.2+K_9)\left[4K_1K_5-(1-K_2-K_4)^2\right]-K_1(K_6+K_8)^2 \\ &-(1-K_2-K_4)(\mu+K_3+K_7)(K_6+K_8)-K_5(\mu+K_3+K_7)^2 > 0. \end{aligned}$$

In Figure 3, the synchronization of two chaotic systems (16) and (17) is presented. The parameters were taken as: $\mu = 1.6, K_1 = K_4 = K_5 = 1, K_2 = 0, K_3 = K_6 = K_8 = \frac{1}{2}, K_9 = 3, K_7 = 0$.

4 Conclusion

In this paper, we apply the generalized Hamiltonian system and observer approach and obtain two chaotic systems: the "transmitter" and the "receiver" dynamics. Practically,

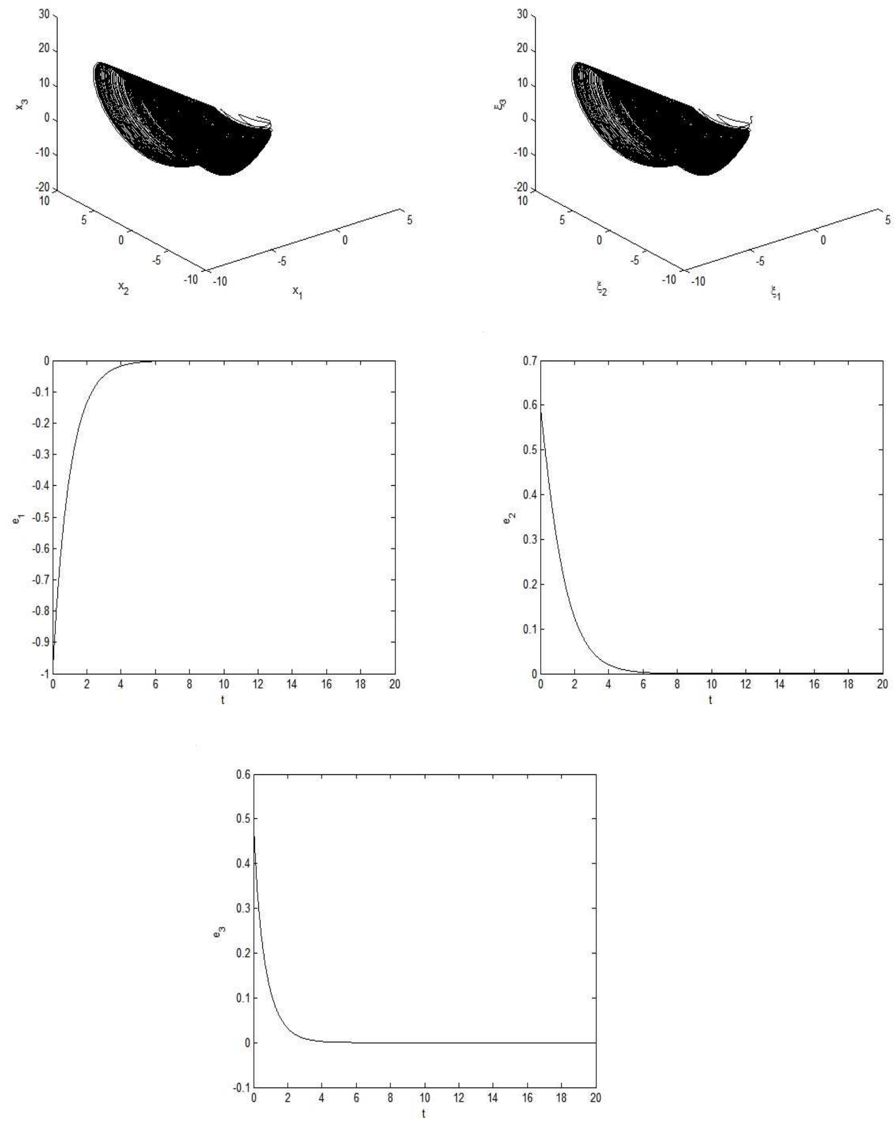

Figure 3: The synchronization of the 3D model (16) and (17).

two chaotic systems are the systems coupled unidirectionally. We study mainly the condition with which the coupling coefficient matrix K is satisfied when the complete synchronization of two coupled chaotic systems happens.

Acknowledgments

The authors acknowledge the helpful advices and suggestions of the reviewers and appreciate the support of National Natural Science Foundation of China (No. 10702065).

References

[1] Boccaletti, S., Kurths, J., Osipov, G., Valladares, D.L. and Zhou, C.S. The synchronization of chaotic systems. *Physics Reports* **366** (2003) 1–101.

[2] Hirokazu, F. and Tomoji, Y. Stability Theory of Synchronized Motion in Coupled-Oscillator Systems. *Progress of Theoretical Physics* **69** (1983) 32–47.

[3] Afraimovich, V.S., Verichev, N.N. and Rabinovich, M.I. Stochastic synchronization of oscillations in dissipative systems. *Izv. Vys. Uch. Zav. Radiofizika* **29** (1986) 1050–1060.

[4] Pecora, L.M. and Carroll, T.L. Synchronization in chaotic system. *Phys. Rev. Lett.* **64** (1990) 821–823.

[5] Rosenblum, M.G., Pikovsky, A.S. and Kurths J. Phase Synchronization of Chaotic Oscillators. *Phys. Rev. Lett.* **76** (1996) 1804–1807.

[6] Rosa, E., Ott, E.J.W. and Hess, M.H. Transition to Phase Synchronization of Chaos. *Phys. Rev. Lett.* **80** (1998) 1642–1645.

[7] Rosenblum, M.G., Pikovsky, A.S. and Kurths, J. From Phase to Lag Synchronization in Coupled Chaotic Oscillators. *Phys. Rev. Lett.* **78** (1997) 4193–4196.

[8] Rulkov, N.F., Sushchik, M.M. and Tsimring, L.S. Generalized synchronization of chaos in directionally coupled chaotic systems. *Phys. Rev. E* **51** (1995) 980–994.

[9] Kocarev, L. and Parlitz, U. Generalized synchronization, predictability, and equivalence of unidirectionally coupled dynamical systems. *Phys. Rev. Lett.* **76** (1996) 1816–1819.

[10] Boccaletti, S. and Valladares, D.L. Characterization of intermittent lag synchronization. *Phys. Rev. E* **62** (2000) 7497–7500.

[11] Zaks, M.A., Park, E.H., Rosenblum M.G. and Kurths,J. Alternating locking ratios in imperfect phase synchronization. *Phys. Rev. Lett.* **82** (1999) 4228–4231.

[12] Femat, R. and Solos-Perales, G. On the chaos synchronization phenomena. *Physics Letters A* **262** (1999) 50–60.

[13] Zanette, D.H. Dynamics of globally coupled bistable elements. *Phys. Rev. E* **55** (1997) 5315–5320.

[14] Boccaletti, S., Bragard, J., Arecchi, F.T. and Mancini, H.L. Synchronization in Nonidentical Extended Systems. *Phys. Rev. Lett.* **83** (1999) 536–539.

[15] Parmananda, P. Generalized synchronization of spatiotemporal chemical chaos. *Phys. Rev. E* **56** (1997) 1595–1598.

[16] Amengual, A., Hernondez-Garcıo, E., Montagne, R. and Miguel, M.S. Synchronization of Spatiotemporal Chaos: The Regime of Coupled Spatiotemporal Intermittency. *Phys. Rev. Lett.* **78** (1997) 4379–4382.

[17] Tass, P., Rosenblum, M.G., Weule, J., Kurths, J., et al. *Phys. Rev. Lett.* **81** (1998) 3291–3294.

[18] Neiman, A., Pei, X., Russell, D., et al. Synchronization of the noisy electrosensitive cells in the paddlefish. *Phys. Rev. Lett.* **82** (1999) 660–663.

[19] Maza, D., Vallone, A., Mancini, H. and S. Boccaletti. Experimental phase synchronization of a chaotic convective flow. *Phys. Rev. Lett.* **85** (2000) 5567–5570.

[20] Hall, G.M., Bahar, S. and Gauthier, D.J. Prevalence of Rate-Dependent Behaviors in Cardiac Muscle. *Phys. Rev. Lett.* **82** (1999): 2995–2998.

[21] Ticos, C.M., Rosa, E., William, J., et al. Experimental real time phase synchronization of a paced chaotic plasma discharge. *Phys. Rev. Lett.* **85** (2000) 2929–2932.

[22] DeShazer, D.J., Breban, R., Ott, E. and Roy, R. Detecting phase synchronization in a chaotic laser array. *Phys. Rev. Lett.* **87** (2001) 044101(1-4).

[23] Sira–Ramoırez, H. and Cruz–Hernoandez, C. Synchronization of chaotic systems: A Gener-alized Hamiltonian systems approach. *Int. J. Bifurcat. Chaos* **11(5)** (2001) 1381–1395.

[24] Cruz–Hernoandez, C. Synchronization of time-delay chua's oscillator with application to secure communication. *Nonlinear Dynamics and Systems Theory* **4**(1) (2004) 1–14.

[25] Sun P. The periodic small disturbance controlling the chaotic motion of the forced brusselator. *Journal of Anshan Institute of I. & S. Technology* **20**(2) (1997) 47–49.

[26] Khalil, H. *Nonlinear systems.* Mecmillan Publishing Company, a division of Macmillan. Inc., 1992.

[27] Lü, L. and Ou, Y.H. Studies on Variable Rate Pulse Feedback of Chaos System. *Chinese Journal of Chemical Physics* **16**(6) (2003) 459–462.