Homoclinic Orbits for Superquadratic Hamiltonian Systems with Small Forcing Terms

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Abstract: In this paper, we prove the existence of homoclinic orbits for the second order Hamiltonian system: \( \ddot{q}(t) + \nabla V(t, q(t)) = f(t) \), where \( V \in C^1(\mathbb{R} \times \mathbb{R}^n, \mathbb{R}) \), \( V(t, q) = -K(t, q) + W(t, q) \) is \( T \)-periodic in \( t \), \( K \) satisfies the "pinching" condition \( b_1|q|^2 \leq K(t, q) \leq b_2|q|^2 \) and \( W \) is superquadratic at the infinity and needs not satisfy the global Ambrosetti-Rabinowitz condition. A homoclinic orbit is obtained as the limit of \( 2kT \)-periodic solutions of a certain sequence of second order differential equations.

Keywords: homoclinic orbit; Hamiltonian system; Mountain Pass Theorem.

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1 Introduction

Let us consider the second order Hamiltonian system

\[ \ddot{q}(t) + \nabla V(t, q(t)) = f(t), \quad (HS) \]

where \( V(t, x) = -K(t, x) + W(t, x), \nabla V(t, x) = (\partial V/\partial x)(t, x), \ K, W : \mathbb{R} \times \mathbb{R}^n \rightarrow \mathbb{R} \) are \( C^1 \)-maps, \( T \)-periodic with respect to \( t \), \( T > 0 \) and \( f : \mathbb{R} \rightarrow \mathbb{R}^n \) is continuous and bounded. We will say that a solution \( q \) of \( (HS) \) is homoclinic (to 0) if \( q(t) \rightarrow 0 \) as \( t \rightarrow \pm \infty \). In addition, if \( q \neq 0 \) then \( q \) is called a nontrivial homoclinic solution.

The problem of finding subharmonic and homoclinic solutions for Hamiltonian systems has been the object of many works under different assumptions on the growth