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A New Interconnected Observer Design in Power Converter: Theory and Experimentation

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Abstract: This paper deals with an observer design for a P-Cell Chopper. The goal is to reduce drastically the number of sensors in such system by using an observer in order to estimate all the capacitor voltages. Furthermore, considering an instantaneous model of a p-cell chopper, an interconnected observer is designed in order to estimate the capacitor voltages. This is realized by using only the load current measurement. Simulation results are given in order to illustrate the performance of such observer. To show the validity of our approach, experiments based an DSP results are presented.

Keywords: *p-cell chopper; observer design; interconnected observer.*

Mathematics Subject Classification (2000): 93C10, 93A15, 93C95.

1 Introduction

The power electronics knows important technological developments. This is carried out thanks to the developments of the power semiconductor but also of new energy conversion systems. Among these systems, Multi-Cell Chopper are based on the association in series of the elementary cells of commutation. This structure, appeared at the beginning of the 90's [20, 18], and makes it possible to share the constraints in tension and also to improve the harmonic contents of the waves forms [10].

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Form the practical point of view, the series multi-cell converter, designed by the LEEI (Toulouse-France), leads to a safe series association of components working in switching mode. This new structure combines additional benefits: attenuation of the voltage jump and modularity of the topologies. All these qualities make this new topology very attractive in many industrial applications. For instance, GEC/ACEC implements this proposal to realize the input chopper which supplies their "T13" locomotives in power. Three-phase inverters called "symphony" and developed by Alstom to drive electric motors are also based on the same principe.

To benefit as well as possible from the large potential of the multicellular structure, researcher went in various directions.

Furthermore, the normal operation of the series p-cell converter is obtained when the voltages are $v_{ci} = iE/p, i = 1, ..., p-1$ (see Figure 2). These voltages are generated when a suitable control of switches is applied in order to obtain a specific value. The control inserted of the switches allows cancelling the harmonics at the switching frequency F_{sw} and reducing the ripple of the chopped voltage. However, these properties are lost if the voltages of these capacitors drift. On the other hand, if a specific control is desired, it is advisable to measure these voltages in order to implement it. But, it is not easy because extra sensors are necessary to measure these voltages, then it increases the cost. For this reason, it should be avoided and the estimation of these voltages becomes an attractive and economical option. It is for such reason that, an original method to eliminate such sensors is the use of observers. From, control theory point of view, an observer is considered as a software sensor used to estimate the unmeasurable variables of a system.

On the other hand, several approaches have been considered to develop new methods of control and observation of the p-cell converter. Initially, models have been developed to describe their instantaneous behaviors [10], harmonic [11] or averaging [1]. These various models were used as the base for the development of control laws in open-loop [18] and in closed-loop [15, 21].

Until now, all these p-cell converters are driven successfully, by means of a fix frequency modulator based on pulse width modulation(PWM). Current control algorithms do not take into account the fact that any power converter is a discrete and discontinuous plant, or, at least a hybrid one. Nevertheless, the profitable skill of PWM technique is to ensure a well-known steady state behavior which is "optimal" for the electric load with respect to harmonic attenuation. Furthermore, some representations of the p-cell converter considered complex models and need to be discretized in order to design a discrete observer to be implemented.

Then, in all proposed methods a considerable number of feedback signals are required which are associated with extra cost of sensing devices. To reduce the cost of sensors, a methodology to estimate the voltages in the capacitors is necessary.

In [3, 4], the observer canonical form consisting of a linear output map and linear dynamics driven by a nonlinear output injection is used. The resulting observer has exactly linear error dynamics, i.e., nonlinearities are compensated exactly. The approaches suggested in [5, 6, 7] rely on the observability canonical form, which has significantly weaker existence conditions than the observer canonical form. In the observability canonical form, the observer is designed by a high-gain technique with a constant observer gain, i.e., the nonlinearities are not compensated but dominated by a linear part. For an implementation of the observer in the original coordinates one gets a Luenberger-like observer with a possibly nonlinear gain vector field [8, 9]. In the last decade, new approaches have been developed for nonlinear systems that are not uniformly observable. Several approaches use Kalman-like decompositions [8].

In this article, we develop an observer for p-cell chopper based on an instantaneous model describing the dynamical behavior of the p-cell converter. This model is constructed in order to design an observer estimating each flying capacitor voltage. The proposed observer design is based on the class of nonlinear systems which can be written in the form of affine state systems, for which the problem of state observer design has been studied. This class of observers is based on the excitation condition in order to guarantee its convergence.

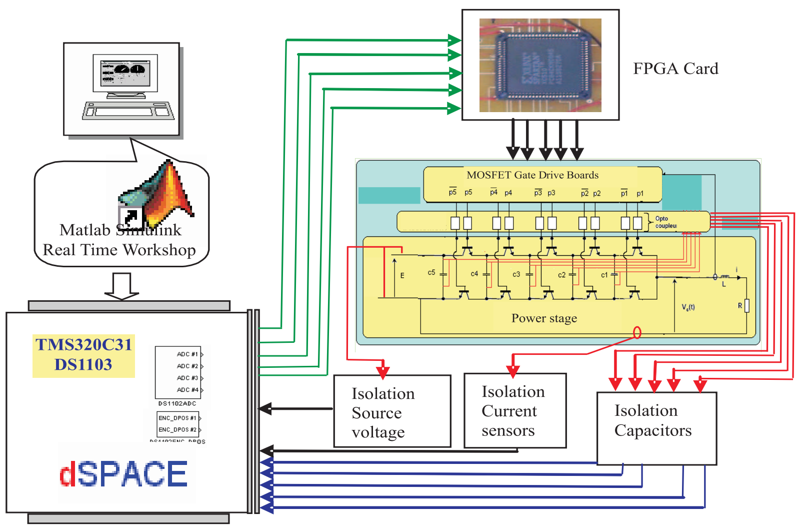

Figure 1: General structure of the dSPACE observer.

The objective of this work is to design an observer for a P-Cell Chopper converter in order to estimate the unmeasurable voltages of the capacitors using the load current i and the voltage of the source E, and give an experimental validation of it. The block diagram describing the proposed observation scheme is illustrated in Figure 1.

The paper is organized as follows: In Section 2, the instantaneous model of P-cell chopper is introduced. In Section 3, the observability properties of the P-Cell Chopper model are given. The observer design based on an new representation of the instantaneous model of the converter is presented in Section 4. In Section 5, using a model of 5 cells chopper, simulations results are shown in order to illustrate the performance of the proposed observer. The proposed observation scheme is validated and experimental results are given. Finally, some conclusions end the paper.

2 P-Cell Converter Model

Throughout the paper, the p-cell converter connects in series p elementary cells and a passive load R and L as illustrated in Figure 2. Each switching cell is controlled by a binary input signal $S_k(t)$ for $k = 1, ..., p$.

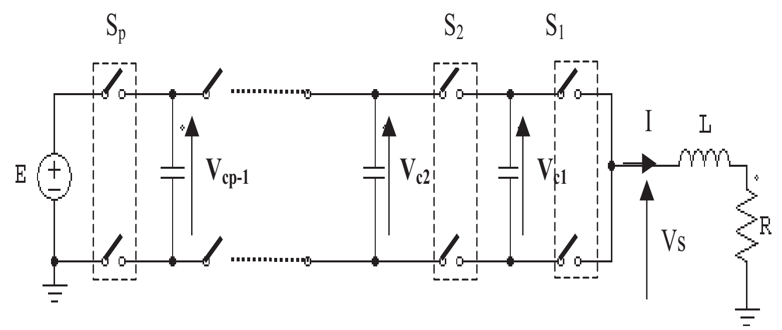

Figure 2: A p-cells converter.

This signal $S_k(t)$ is equal to 1 when the upper switch of the cell is conducting and to 0 when the lower complementary switch of the cell is conducting. The mathematical model describing the behavior of a p-cell converter is given by

$$\Sigma_{pcell}:\left\{\begin{array}{rcl} \frac{dI}{dt} &=& -\frac{R}{L}I+\frac{E}{L}S_p-\frac{v_{c_{p-1}}}{L}\left(S_p-S_{p-1}\right)\ldots-\frac{v_{c_1}}{L}\left(S_2-S_1\right), \\ \frac{dv_{c_1}}{dt} &=& \frac{1}{c_1}\left(S_2-S_1\right)I, \\ \frac{dv_{c_2}}{dt} &=& \frac{1}{c_2}\left(S_3-S_2\right)I, \\ &\vdots& \\ \frac{dv_{c_{p-1}}}{dt} &=& \frac{1}{c_p}\left(S_p-S_{p-1}\right)I, \\ y &=& I, \end{array}\right. \tag{1}$$

where v_{c_k} is the k^{th} flying capacitor voltage and I is the output load current, and is the only measurable output. c_k for $k = 1, ..., p$; are the capacitors, E is the voltage of the source, R is the resistance and L is the inductance.

Now, from the instantaneous state model of the p-cell converter given in (1), we will analyze the observability properties of such system in order to construct an observer. It is well known that the observability of nonlinear systems depends on the applied input, and a study of the different classes of inputs which render the system observable or unobservable is given in [16, 17].

Rewriting the model (1) in the state affine form, we have:

$$\Sigma:\left\{\begin{array}{l} \dot{X}=\bar{A}(u)X+\bar{B}(u), \\ y=\bar{C}X, \end{array}\right. \tag{2}$$

where $X = (I, v_{c_1}, ..., v_{c_{p-1}})$ is the state vector, $u = \{S_1, ..., S_p\}$ is the input sequence applied to the converter,

$$\bar{A}(u) = \begin{pmatrix} -\frac{R}{L} & -\frac{1}{L}(S_2 - S_1) & \dots & -\frac{1}{L}(S_p - S_{p-1}) \\ -\frac{1}{c_1}(S_2 - S_1) & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ -\frac{1}{c_{p-1}}(S_p - S_{p-1}) & 0 & \dots & 0 \end{pmatrix},$$

$$\bar{B}(u) = \begin{pmatrix} \frac{E}{L}S_p \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \quad \bar{C} = (1, 0, \dots, 0).$$

Regarding the instantaneous model of the multi-cell converter (2), we can see that there are several operating switching modes (S_k, S_{k+1}) which render the system unobservable, i.e. for the following operating switching modes

$$(S_k, S_{k+1}) = (1,1) \quad \text{and} \quad (S_k, S_{k+1}) = (0,0), \quad \text{for} \quad k = 1, ..., p-1,$$

the system becomes unobservable. These operating switching modes are not affected by the capacitor voltage. However, these cases occur only for a part of control sequence. If it occurs for all the control sequences this is not of physical interest because they represent particular situations in which the cell chopper is not operating.

Now, the sequence of corresponding input $u = \{u_1, ..., u_{p-1}\}$, where $u_k = S_{k+1} - S_k$, applied to the system (1), is sufficiently periodic. Furthermore, assuming that the current I is the only measurable variable of the system (2), from the observability rank condition, it follows that

$$\text{Rank}\left(\ \bar{C}, \bar{C}\bar{A}(u), ..., \bar{C}\bar{A}^{p-1}(u) \ \right)^T = 2. \tag{3}$$

It is clear that the system is not of full rank, i.e. the system is not observable. Then, in order to overcome this difficulty we consider a new representation of the multi-cell converted which is constituted of a set of subsystems of dimension 2. These subsystems are such that the whole system is represented as an interconnected structure. Furthermore, an analysis of the observability of each subsystem is required and is given in the next section.

3 Observer Design for a P-Cell Chopper

Now, in this section, the design of $p-1$ interconnected observers for p- cell chopper is given. For that, we will consider a different representation of system (1) such that the original system can be splitted into a suitable set of $p-1$ subsystems for which it will be possible to design an observer for estimating the capacitor voltages v_{c_j}, for $j = 1, ..., p-1$.

Next, considering that system (1) can be splitted into $p-1$ interconnected subsystems of the form

$$\Sigma_k : \begin{cases} \frac{dI}{dt} &= -\frac{R}{L}I + \frac{E}{L}S_P - \frac{1}{L}\sum\limits_{j=1}^{p-1}(S_{j+1} - S_j)\, v_{c_j}, \\ \frac{dv_{c_k}}{dt} &= \frac{1}{c_k}(S_{k+1} - S_k)\, i, \\ y &= I, \end{cases}$$

where the above system can be represented, for $k=1,...,p-1$, in a compact form as:

$$\Sigma_k : \begin{cases} \dot{X}_k = A_k\left(u_k\right)X_k + B_k(\bar{u}_k, \bar{X}_k), \\ y = \mathbf{C}_k X_k, \end{cases} \tag{4}$$

where $X_k = \left(I,\ v_{c_k}\right)^T$ is the state vector of subsystem (4), $X = \left(I, v_{c_1}, ..., v_{c_{p-1}}\right)^T$ is the state of system (1), $\bar{X}_k = (v_{c_1}, ..., v_{c_{k-1}}, v_{c_{k+1}}, ..., v_{c_{p-1}})^T$, $u_k = S_{k+1} - S_k$, for $k = 1, ..., p-1$; and $\bar{u}_k = \left(u_1, ..., u_{k-1}, u_{k+1}, ..., u_p\right)^T$, are the inputs. Furthermore, $y = \mathbf{C}_k X_k = I$ is the output of subsystem (4) with $\mathbf{C}_k = (1\ 0)$ for $k = 1, ..., p-1$; and

$$A_k\left(u_k\right) = \begin{pmatrix} -\frac{R}{L} & -\frac{(u_k)}{L} \\ \frac{(u_k)}{c_k} & 0 \end{pmatrix}, \tag{5}$$

$$B_k(\bar{u}_k, \bar{X}_k) = \begin{pmatrix} -\frac{1}{L}\sum\limits_{j=1,\ j\neq k}^{p-1}(S_{j+1}-S_j)v_{c_j} + \frac{E}{L}S_p \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{L}\bar{u}_k^T\bar{X}_k + \frac{E}{L}S_p \\ 0 \end{pmatrix}. \tag{6}$$

It is clear that for $u_k = 0$, the system becomes unobservable. However, each subsystem k^{th}, which is of dimension 2, is observable for an appropriate input u_k and its rank is equal to 2. Furthermore, in order to estimate the unmeasurable variables, no feedback is applied to excite the converter as it has been proposed in other works. Instead of this, we consider an equivalent concept which is the well-known concept of regularly persistent input (see Appendix). More precisely, a regularly persistent input applied to the system allows to excite the system sufficiently to obtain the information necessary to be able to reconstruct the unmeasurable variables by means of an observer. If the input is not sufficiently persistent, then it is not possible to reconstruct the state of the system from the measured output and the applied input.

Furthermore, the function $B_k(\bar{u}_k, \bar{X}_k)$ is the interconnection term depending on inputs and states of each subsystem. Notice that the output is the current $I(t)$ and is the same for each subsystem. Then, the following system

$$O_k : \begin{cases} \dot{Z}_k = A_k\left(u_k\right)Z_k + B_k(\bar{u}_k, \bar{Z}_k) - P_k^{-1}\mathbf{C}_k^T\left(y_k - \hat{y}_k\right), \\ \dot{P}_k = -\theta_k P_k - A_k^T(u_k)P_k - P_k A_k(u_k) + \mathbf{C}_k^T\mathbf{C}_k, \end{cases} \tag{7}$$

is an observer for subsystem (4), for $k = 1, 2, ..., p-1$; where $\theta_k > 0$, $\hat{y}_k = \mathbf{C}_k X_k = \hat{I}$ and $P_k^{-1}\mathbf{C}_k^T$ is the gain of the observer which depends on the solution of the second equation of (7) for each subsystem with $Z_k = (\hat{I},\ \hat{v}_{c_k})^T$, $\bar{Z}_k = (\hat{v}_{c_1}, ..., \hat{v}_{c_{k-1}}, \hat{v}_{c_{k+1}}, ..., \hat{v}_{c_{p-1}})^T$ and $A_k\left(u_k\right)$ is given in (5) and for $k = 1, 2, ..., p-1$;

$$B_k(\bar{u}_k, \bar{Z}_k) = \begin{pmatrix} -\frac{1}{L}\sum\limits_{j=1,\ j\neq k}^{p-1}(S_{j+1}-S_j)\hat{v}_{c_j} + \frac{E}{L}S_p \\ 0 \end{pmatrix} = \begin{pmatrix} -\frac{1}{L}\bar{u}_k^T\bar{Z}_k + \frac{E}{L}S_p \\ 0 \end{pmatrix}.$$

Now, consider that system (1) can be represented as follows:

$$\Sigma : \begin{cases} \dot{X}_1 = A_1\left(u_1\right)X_1 + B_1(\bar{u}_1, \bar{X}_1), \\ \dot{X}_2 = A_2\left(u_2\right)X_2 + B_2(\bar{u}_2, \bar{X}_2), \\ \vdots \\ \dot{X}_{p-1} = A_{p-1}\left(u_{p-1}\right)X_{p-1} + B_{p-1}(\bar{u}_{p-1}, \bar{X}_{p-1}). \end{cases} \tag{8}$$

Notice that the output is the current $I(t)$ and is the same for each subsystem. The main idea of the paper is to construct an observer for the whole system (1), from the separate observer design of each subsystem (4).

In general, if each (7) is an exponential observer for (4), for $k = 1, 2, ..., p-1$; then the following interconnected system

$$O : \begin{cases} \dot{Z}_1 = A_1(u_1) Z_1 + B_1(\bar{u}_1, \bar{Z}_1) - P_1^{-1} \mathbf{C}_1^T (y - \hat{y}), \\ \dot{Z}_2 = A_2(u_2) Z_2 + B_2(\bar{u}_2, \bar{Z}_2) - P_2^{-1} \mathbf{C}_2^T (y - \hat{y}), \\ \vdots \\ \dot{Z}_{p-1} = A_{p-1}(u_{p-1}) Z_{p-1} + B_{p-1}(\bar{u}_{p-1}, \bar{Z}_{p-1}) - P_{p-1}^{-1} \mathbf{C}_{p-1}^T (y - \hat{y}), \\ \dot{P}_1 = -\theta_1 P_1 - A_1^T(u_1) P_1 - P_1 A_1(u_1) + \mathbf{C}_1^T \mathbf{C}_1, \\ \dot{P}_2 = -\theta_2 P_2 - A_2^T(u_2) P_2 - P_2 A_2(u_2) + \mathbf{C}_2^T \mathbf{C}_2, \\ \vdots \\ \dot{P}_{p-1} = -\theta_{p-1} P_{p-1} - A_{p-1}^T(u_{p-1}) P_{p-1} - P_{p-1} A_{p-1}(u_{p-1}) + \mathbf{C}_{p-1}^T \mathbf{C}_{p-1}, \end{cases} \tag{9}$$

is an observer for the interconnected system (8).

Remark 3.1 The proposed observer 9 works for inputs satisfying the regularly persistent condition, which is equivalent to each subsystem (4) being observable, and hence, observer (7) works at the same time while the other subsystems become observable when their corresponding input satisfies the regularly persistent condition.

Now, we will give the sufficient conditions which ensure the convergence of the interconnected observer (9). For that, we introduce the following assumptions.

Assumption 3.1 Assume that the input $u_k = S_{k+1} - S_k$, for $k = 1, 2, ..., p-1$; is regularly persistent input for subsystem (4), and admits an exponential observer (7). The estimation error, defined as $\varepsilon_k = Z_k - X_k$, is bounded.

Assumption 3.2 The term $B_k(\bar{u}_k, \bar{X}_k)$ does not destroy the observability property of the subsystem (4), under the action of the regularly persistent input $u_k = (S_{k+1} - S_k)$, for $k = 1, 2, ..., p-1$. Moreover, $B_k(\bar{u}_k, \bar{X}_k)$ is Lipschitz with respect to $\bar{X}_k$ and uniform with respect to $\bar{u}_k$, for $k = 1, 2, ..., p-1$.

The observer convergence can be proved only if the inputs u_k are regularly persistent, i.e. it is a class of admissible inputs that allows to observe the system (for more details see [19, 20]). This assumption guarantees that the observer works and that its gain is well-defined, i.e. the matrices P_k, for $k = 1, 2, ..., p-1$, are nonsingular (see appendix).

The following result can be established.

Proposition 3.1 *Consider the system (1) can be represented in the form of system (8), where each subsystem (4) satisfies the assumptions 3.1 and 3.2, for $k = 1, 2, ..., p-1$. Then, system (9) is an exponential observer for system (8). Furthermore, the estimation error, defined as $\varepsilon = Z - X$, converges exponentially to zero.*

Proof In order to prove the convergence of the observer (9), first we consider the dynamics of subsystem (4), for which an observer of the form (7) can be designed. Then, defining the estimation error $\varepsilon_k = Z_k - X_k$ whose dynamics is given by

$$\dot{\varepsilon}_k = \{A(u_k) - P_k^{-1}\mathbf{C}_k^T\mathbf{C}_k\}\varepsilon_k + \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) \tag{10}$$

for $k = 1, \dots, p-1$; where $\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) = B_k(\bar{u}_k, \bar{Z}_k) - B_k(\bar{u}_k, \bar{X}_k)$.

From Assumption 3.1 and Lemma 7.1 (see Appendix), we can define $V = \sum_{l=1,}^{p-1} V_k$ as a Lyapunov function for the interconnected system (8), where $V(\varepsilon_k) = \varepsilon_k^T P_k \varepsilon_k$ is a Lyapunov function for subsystem (4). It is clear that these functions are well defined because the matrices P_k are nonsingular.

Taking the time derivative of $V(\varepsilon_k)$, it follows that

$$\dot{V}(\varepsilon_k) \leq -\theta_k V(\varepsilon_k) + \varepsilon_j^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) \quad \text{for} \quad k = 1, \dots, p-1. \tag{11}$$

Now, adding and subtracting the term $\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)$, we have

$$\dot{V}(\varepsilon_k) \leq -\theta_k V(\varepsilon_k) + 2\varepsilon_k^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) \pm \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k).$$

Next, regrouping the appropriate terms gives:

$$\begin{aligned} \dot{V}(\varepsilon_k) \quad \leq \quad & -(\theta_k - 1)\|\varepsilon_k\|_{P_k}^2 \\ & -\|\varepsilon_k\|_{P_k}^2 + 2\varepsilon_k^T P_k \Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k) - \|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|_{P_k}^2 \\ & +\|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|_{P_k}^2. \end{aligned} \tag{12}$$

It follows that

$$\dot{V}(\varepsilon_k) \leq -(\theta_k - 1)\|\varepsilon_k\|_{P_k}^2 + \|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|_{P_k}^2. \tag{13}$$

Now, from assumption 3.2, $B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)$ is Lipschitz, it follows that

$$\|\Delta B_k(\bar{u}_k, \bar{X}_k, \bar{Z}_k)\|_{P_k}^2 < \sum_{l=1, l\neq k}^{p-1} \lambda_l \|\varepsilon_l\|_{P_k}^2. \tag{14}$$

we get:

$$\dot{V}(\varepsilon_k) \leq -(\theta_k - 1)\|\varepsilon_k\|_{P_k}^2 + \lambda_l \|\varepsilon_l\|_{P_k}^2, \tag{15}$$

the time derivative of V is given by

$$\dot{V}(\varepsilon) = \sum_{k=1,}^{p-1} \dot{V}(\varepsilon_k), \tag{16}$$

$$\dot{V}(\varepsilon) \leq \sum_{k=1,}^{p-1} \Big\{ -(\theta_k - 1)\|\varepsilon_k\|_{P_k}^2 + \sum_{l=1, l\neq k}^{p} \lambda_l \|\varepsilon_l\|_{P_k}^2 \Big\}. \tag{17}$$

Using the lemma on equivalence of norms, i.e. there exists a positive constant μ_l such that $\|\varepsilon_l\|_{P_k}^2 \leq \mu_l \|\varepsilon_l\|_{P_l}^2, \forall l = 1, ..., p-1$. Then, it follows that

$$\dot{V}(\varepsilon) \leq \sum_{k=1,}^{p-1} \Big\{ -(\theta_k - 1)\|\varepsilon_k\|_{P_k}^2 + \sum_{l=1, l\neq k}^{p-1} \lambda_l \mu_l \|\varepsilon_l\|_{P_k}^2 \Big\} \tag{18}$$

or

$$\dot{V}(\varepsilon) \leq \sum_{k=1}^{p-1} - \left\{(\theta_k - 1) - (p-1)\lambda_l \mu_l\right\} \|\varepsilon_j\|^2_{P_k}. \tag{19}$$

Finally, we have $V(\varepsilon) \leq V(\varepsilon(t_0))e^{-\gamma(t-t_0)}$, for $\gamma = min\,(\gamma_1, ..., \gamma_{p-1})$ where $\gamma_k = (\theta_k - 1) - (p-1)\lambda_k\mu_k)$. Taking $\varepsilon = col(\varepsilon_1, ..., \varepsilon_{p-1})$, it is easy to see that

$$\|\varepsilon(t)\| \leq K\|\varepsilon(t_0)\|e^{-\gamma(t-t_0)}. \tag{20}$$

This ends the proof.

4 Observer for 5-Cell Chopper

Now, in this section we present the proposed methodology which is applied to a model of 5-Cell Chopper converter. For that, consider the following model of 5-cell chopper:

$$\Sigma_{5cell}: \left\{ \begin{array}{rcl} \frac{dI}{dt} &=& -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2-S_1)}{L}v_{c_1} - \frac{(S_3-S_2)}{L}v_{c_2} - \frac{(S_4-S_3)}{L}v_{c_3} - \frac{(S_5-S_4)}{L}v_{c_4}, \\ \frac{dv_{c_1}}{dt} &=& \frac{1}{c_1}\left(S_2 - S_1\right) I, \\ \frac{dv_{c_2}}{dt} &=& \frac{1}{c_2}\left(S_3 - S_2\right) I, \\ \frac{dv_{c_3}}{dt} &=& \frac{1}{c_3}\left(S_4 - S_3\right) I, \\ \frac{dv_{c_4}}{dt} &=& \frac{1}{c_4}\left(S_5 - S_4\right) I. \end{array} \right. \tag{21}$$

Following the ideas of this original methodology, the model can be rewritten in the following form:

$$\Sigma_1: \left\{ \begin{array}{rcl} \frac{dI}{dt} &=& -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2-S_1)}{L}v_{c_1} - \frac{(S_3-S_2)}{L}v_{c_2} - \frac{(S_4-S_3)}{L}v_{c_3} - \frac{(S_5-S_4)}{L}v_{c_4}, \\ \frac{dv_{c_1}}{dt} &=& \frac{1}{c_1}\left(S_2 - S_1\right) I, \end{array} \right.$$

$$\Sigma_2: \left\{ \begin{array}{rcl} \frac{dI}{dt} &=& -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2-S_1)}{L}v_{c_1} - \frac{(S_3-S_2)}{L}v_{c_2} - \frac{(S_4-S_3)}{L}v_{c_3} - \frac{(S_5-S_4)}{L}v_{c_4}, \\ \frac{dv_{c_2}}{dt} &=& \frac{1}{c_2}\left(S_3 - S_2\right) I, \end{array} \right.$$

$$\Sigma_3: \left\{ \begin{array}{rcl} \frac{dI}{dt} &=& -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2-S_1)}{L}v_{c_1} - \frac{(S_3-S_2)}{L}v_{c_2} - \frac{(S_4-S_3)}{L}v_{c_3} - \frac{(S_5-S_4)}{L}v_{c_4}, \\ \frac{dv_{c_3}}{dt} &=& \frac{1}{c_3}\left(S_4 - S_3\right) I, \end{array} \right.$$

$$\Sigma_4: \left\{ \begin{array}{rcl} \frac{dI}{dt} &=& -\frac{R}{L}I + \frac{E}{L}S_5 - \frac{(S_2-S_1)}{L}v_{c_1} - \frac{(S_3-S_2)}{L}v_{c_2} - \frac{(S_4-S_3)}{L}v_{c_3} - \frac{(S_5-S_4)}{L}v_{c_4}, \\ \frac{dv_{c_4}}{dt} &=& \frac{1}{c_4}\left(S_5 - S_4\right) I. \end{array} \right.$$

This set of subsystems can be represented in an interconnected compact form as follows

$$\Sigma_i: \left\{ \begin{array}{l} \dot{X}_i = A_i(u_i)X_i + B_i(\bar{u}_i, \bar{X}_i), \\ y = \mathbf{C}_i X_i = I, \end{array} \right. \qquad \text{for} \quad i = 1, ..., 4.$$

It can be assumed that the control sequence of inputs provides the sufficient persistency to guarantee that the observer works correctly (see appendix and assumption 3.1). Using this assumption, an observer for the above interconnected subsystems is given by

$$O_i: \left\{ \begin{array}{l} \hat{Z}_i = A_i(u_i)\hat{X}_i + B_i(\bar{u}_i, \bar{Z}_i) + P_i^{-1}\mathbf{C}_i^T(y - \hat{y}), \\ \dot{P}_i = -\theta_i P_i - A_i^T(u_i)P_i + P_i A_i(u_i) + \mathbf{C}_i^T \mathbf{C}_i, \end{array} \right. \qquad \text{for} \quad i = 1, ..., 4.$$

5 Experimental Results

In this section, we show some experimental results obtained by using the proposed interconnected observer. In order to illustrate the performance of this observer, where the estimated states converge to the real states, the instantaneous converter model of 5 cells (21) is used for the observer design, where the capacitor voltages are estimated. The parameters of the model were chosen as follows:

$$f_d = 16kHz, \quad C = 40\mu F, \quad L = 1mH, \quad R = 100\Omega, \quad E = 120V.$$

Furthermore, to carry out the experimentation and show the efficiency of the proposed observer, we use a trajectory for the input voltage as given in Figure 3.

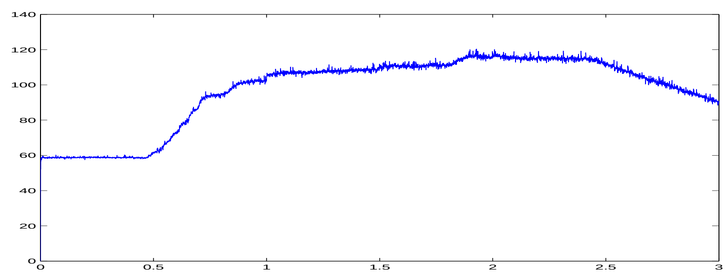

Figure 3: The input voltage E.

Finally, the following initial conditions of the system and the observer were selected as follows. For the system: $X_k = (i, v_{c_k})^T = (0,0)^T$ and for the observer: $Z_k = (\hat{\imath}, \hat{v}_{c_k})$ are given as $(1,20)$, $(1,30)$, $(1,35)$ and $(1,40)$, for $k = 1, ..., 4$. The parameters θ_k, for $k = 1, ..., 4$, which are the design parameters used to control the rate of convergence of each observer, were chosen as follows: $\theta_1 = 30$, $\theta_2 = 40$, $\theta_3 = 50$ and $\theta_4 = 60$.

5.1 Benchmark observation

The experimental setup realized based on the DS1103 dSPACE kit shown in Figure 1 gives the global scheme of the experimental setup. This kit allows real time implementation of converter, it includes several functions such as analog/digital converters and digital signal filtering. In order to run the application we must write our algorithm in C language. Then, we use the RTW and RTI packages to compile and load the algorithm on processor. To visualize and adjust the control parameters in real time we use the software control-desk which allows conducting the process by the computer.

The multi-cells chopper power stage is based on the use of MOSFET. The pulsewidth-modulator (PWM) blocks are generated by FPGA card. The observer is first designed in Simulink/Matlab, then, the Real-Time Workshop is used to automatically generate optimized C code for real time applications. Afterward, the interface between Simulink/Matlab and the digital signal processor (DSP) (DS1103 of dSpace) allows the control algorithm to be run on the hardware.

The master bit I/O is used to generate the required 5 gate signals, and six analog-to-digital converters (ADCs) are used for the sensed line-currents, capacitors voltage, and output voltages. An optical interface board is also designed in order to isolate the entire DSP master bit I/O and ADCS.

5.2 Experimental evaluation

The experimental results of Figures 4–8 are obtained under the following test conditions: The sample time was chosen equal to 50 micro-seconds, and the data acquisition is close to 1 sec in this experimental evaluation. We assume that all parameter are known.

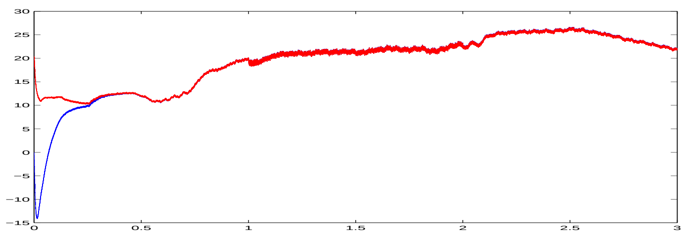

Figure 4: Capacitor voltage V_{c1} measured and estimated.

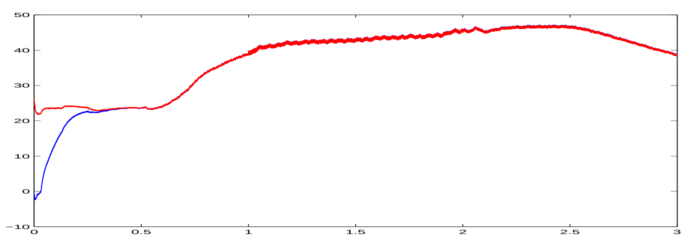

Figure 5: Capacitor voltage V_{c2} measured and estimated.

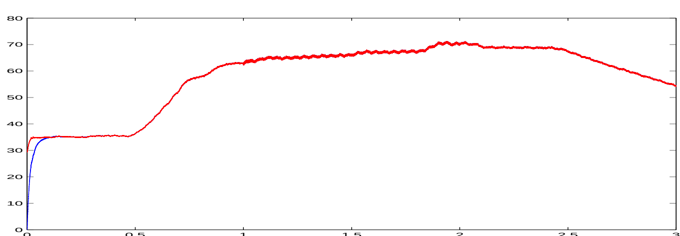

Figure 6: Capacitor voltage V_{c3} measured and estimated.

In order to compare the real and estimated voltages 4 sensors were used, an optical interface was used in this case. Furthermore, to reduce the noise in the signals, a low pass filter was required. In Figures 4–7, we can see the convergence of the estimated and real voltage given by the observer to the real variables, this highlights the well fader performance of the proposed observation scheme. From these plots, we can see that substantial transient of the voltages estimated, is due to the error in the initial conditions. However, these transients can be reduced choosing suitable initial conditions of the observer. In this experiment, the initial conditions were chosen far of them of

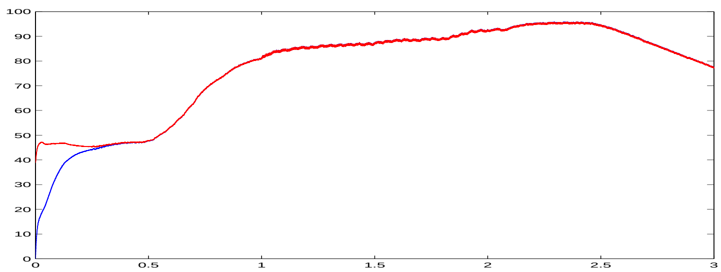

Figure 7: Capacitor voltage V_{c4} measured and estimated.

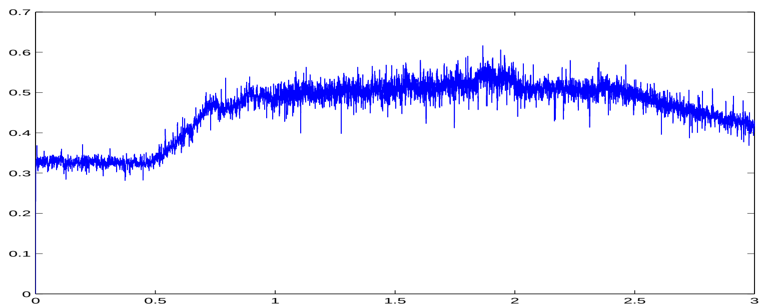

Figure 8: The output current load.

the converter to show the performance of the observer. The output current i is given in Figure 8.

Note that all experimental results are obtained by using a second order filter.

6 Conclusion

In this paper, using an instantaneous model of a Multi-Cell converter, an original methodology of observation has been presented. An observer design has been presented and validated experimentally, to estimate the capacitor voltages from the instantaneous measurement of the current. The practical interest of such observer has been illustrated by means of experimental results. Furthermore, sufficient conditions has been given in order to prove the exponential convergence to zero, with an arbitrary rate of convergence, of the proposed interconnected observer, which only depends on the persistence of the switching control sequence.

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7 Appendix: Some Mathematical Preliminaries

We introduce some definitions related to the inputs applied to the system. Consider a state-affine controlled system of the following form

$$\dot{x} = A(v)x + B(v), \quad y = Cx,$$

where $x \in \mathbf{R}^n; v \in \mathbf{R}^m; y \in \mathbf{R}^p$ with $A : R^m \to \mathcal{M}(n,m); B : R^m \to \mathcal{M}(n,1)$ continuous, and $\mathcal{C} \in \mathcal{M}(p,n)$, where $\mathcal{M}(k,l)$ denotes the space of $k \times l$ matrices with coefficients in R; k (resp.l) is the number of rows (resp. columns).

Notation. Let $\Phi_v(\tau,t)$ denote the transition matrix of:

$$\frac{d}{dt}\Phi_v(\tau,t) = A(v(\tau))\Phi_v(\tau,t), \quad \Phi_v(t,t) = \mathbf{I},$$

with the classical relation: $\Phi_v(t_1,t_2)\Phi_v(t_2,t_3) = \Phi_v(t_1,t_3)$. We then define:

- The *Observability Grammian*: $\Gamma(t,T,v) = \int_t^{t+T} \Phi_v^T(\tau,t)C^T C\Phi_v(\tau,t)d\tau$.
- The *Universality index*: $\gamma(t,T,v) = \min_i(\lambda_i(\Gamma(t,T,v)))$, where the $\lambda_i(M)$ stand for the eigenvalues of a given matrix M.

The input functions are assumed to be measurable and such that $A(v)$ is bounded on the set of admissible inputs of R^+. We recall below some required results of input functions ensuring the existence of an observer for (4).

Definition 7.1 (Regular Persistence). A measurable bounded input v is said to be regularly persistent for the state-affine system (4) if there exist $T > 0; \alpha > 0$ and $t_0 > 0$ such that $\gamma(t,T,v) > \alpha$ for every $t \geq t_0$.

Now, a further result based on regular persistence is introduced.

Lemma 7.1 *Assume that the input u_k is regularly persistent for system (2) and consider the following Lyapunov differential equation:*

$$\dot{P}_k = -\theta_k P_k - A^T(u_k)P_k - P_k A(u_k) + \mathcal{C}_k^T \mathcal{C}_k \tag{22}$$

with $P_k(0) > 0$. Then, $\exists\theta_{k0} > 0$ such that for any symmetric positive definite matrix $P_k(0)$, $\exists\theta_k \geq \theta_{k_0}$, $\exists\alpha_k, \beta_k > 0, t_0 > 0 : \forall t > t_0,\ \alpha_k I < P_k(t) < \beta_k I$, where I is the identity matrix.