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Gradient Transformation Trajectory Following Algorithms for Equality-Constrained Minimization †

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Abstract: For minimizing a scalar-valued function subject to equality constraints, we develop and investigate a family of gradient transformation differential equation algorithms. This family includes, as special cases: Min-Max Ascent, Hestenes' Method of Multipliers, Newton's method, and a Gradient Enhanced Min-Max (GEMM) algorithm that we extend to handle equality constraints. We apply these methods to Rosenbrock's function with a parabolic constraint. We show that Min-Max Ascent is locally and (experimentally) globally asymptotically stable but extremely stiff and has extremely slow convergence. Hestenes' Method of Multipliers is also locally and (experimentally) globally asymptotically stable and has faster convergence, but is still very stiff. Newton's method is not stiff, but does not yield global asymptotic stability. However, GEMM is both globally asymptotically stable and not stiff. We study the stiffness of the gradient transformation family in terms of Lyapunov exponent time histories. Starting from points where all the methods in this paper do work, we show that Min-Max Ascent and Hestenes' Method of Multipliers are very stiff and slow to converge, but with the Method of Multipliers being approximately 2 times as fast as Min-Max Ascent. Newton's method is not stiff and is approximately 900 times as fast as Min-Max Ascent and 400 times as fast as the Method of Multipliers. In contrast, the Gradient Enhanced Min-Max method is globally convergent, is not stiff, and is approximately 100 times faster than Newton's method, 40,000 times faster than the Method of Multipliers, and 90,000 times faster than Min-Max Ascent.

Keywords: *nonlinear programming, Lagrangian Min-Max, stiff differential equations, Lyapunov exponents.*

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† In Memorium: Thomas Lange Vincent, Doctor of Philosophy, Professor Emeritus, The University of Arizona: Born, September 16, 1935; Died, October 26, 2009.

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