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On the Dynamics of a Class of Darwinian Matrix Models†

J. M. Cushing *

Department of Mathematics and the Interdisciplinary Program in Applied Mathematics
University of Arizona, 617 N. Santa Rita, Tucson, AZ 85721 USA

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Abstract: Using the methodology of evolutionary game theory (EGT), I study a class of Darwinian matrix models which are derived from a class of nonlinear matrix models for structured populations that are known to possess stable (normalized) distributions. Utilizing the limiting equations that result from this ergodic property, I prove extinction and stability results for the limiting equations of the EGT versions of these kinds of structured population models. This is done in a bifurcation theory context. The results provide conditions sufficient for a branch of non-extinction equilibria to bifurcate from the branch of extinction equilibria. When this bifurcation is supercritical (explicit criteria are given), these equilibria are stable and represent stable non-extinction equilibria (which are also candidate ESS equilibria). These kinds of matrix models are motivated by applications to size structured populations, and I give an application of this type. Besides illustrating the formal theory, this application shows the importance of trade-offs among life history parameters that are necessary for the existence of an evolutionarily stable equilibrium.

Keywords: *structured population dynamics; nonlinear matrix model; stable distribution; limiting equation; evolutionary game theory; bifurcation; equilibrium, stability.*

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