On the Dynamics of a Class of Darwinian Matrix Models

J. M. Cushing

Department of Mathematics and the Interdisciplinary Program in Applied Mathematics
University of Arizona, 617 N. Santa Rita, Tucson, AZ 85721 USA

Received: November 15, 2009; Revised: March 24, 2010

Abstract: Using the methodology of evolutionary game theory (EGT), I study a class of Darwinian matrix models which are derived from a class of nonlinear matrix models for structured populations that are known to possess stable (normalized) distributions. Utilizing the limiting equations that result from this ergodic property, I prove extinction and stability results for the limiting equations of the EGT versions of these kinds of structured population models. This is done in a bifurcation theory context. The results provide conditions sufficient for a branch of non-extinction equilibria to bifurcate from the branch of extinction equilibria. When this bifurcation is supercritical (explicit criteria are given), these equilibria are stable and represent stable non-extinction equilibria (which are also candidate ESS equilibria). These kinds of matrix models are motivated by applications to size structured populations, and I give an application of this type. Besides illustrating the formal theory, this application shows the importance of trade-offs among life history parameters that are necessary for the existence of an evolutionarily stable equilibrium.

Keywords: structured population dynamics; nonlinear matrix model; stable distribution; limiting equation; evolutionary game theory; bifurcation; equilibrium, stability.

Mathematics Subject Classification (2000): 92D15, 92D25, 39A60.

1 Introduction

Nonlinear matrix models are widely used to describe and study the discrete time dynamics of structured populations. These models take the form

\[ x(t+1) = P(x(t))x(t), \]

where

\[ P(x(t)) = \begin{pmatrix} p_{11}(x(t)) & p_{12}(x(t)) \\ p_{21}(x(t)) & p_{22}(x(t)) \end{pmatrix} \]

and

\[ p_{ij}(x(t)) = \frac{d}{dx} \left( p_{ij}(x) \right) \bigg|_{x=x(t)} \]