The Boundedness of Solutions to Nonlinear Third Order Differential Equations

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Abstract: In this paper, we establish some new sufficient conditions under which all solutions of nonlinear third order differential equations of the form

\[ x''' + \psi(x, x')x'' + f(x, x') = p(t, x, x', x'') \]

are bounded. For illustrations, an example is also given on the bounded solutions.

Keywords: nonlinear differential equations; third order; boundedness of solutions; Lyapunov’s second method.

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1 Introduction

In a recent paper, Omeike [5] considered the following nonlinear third order differential equation:

\[ x''' + \psi(x, x')x'' + f(x, x') = 0. \]  (1.1)

He introduced a Lyapunov function and then discussed the global asymptotic stability of zero solution \( x(t) = 0 \) of this equation. By this work, the author proved under less restrictive conditions the stability result obtained by Qian [6] for equation (1.1). The Lyapunov function introduced in that paper, [5], raised this case. It should be noted that, first in 1970, Barbashin [2] proved some results related to the qualitative behaviors of solution of some systems of third order differential equation. Later, based on the results of Barbashin [2], some results have been improved concerning the qualitative behaviors of

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