A Short Note on Semilinear Elliptic Equations in Unbounded Domain

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Abstract: Let $\Omega \subset \mathbb{R}^n$ be a domain (not necessarily bounded) with smooth boundary $\partial \Omega$. Let $1 \leq n \leq 6$ and $f \in C^{0,\alpha}(\overline{\Omega}) \cap L^2(\Omega)$ be a given function with $f < 0$. In the present study, we prove that the following BVP

$$-\Delta u = u^2 + f \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial \Omega,$$

has a solution $u \in H^1_0(\Omega)$ and satisfies $u \leq 0$ in $\Omega$.

Keywords: monotone iteration method; maximum principle; unbounded domain.


1 Introduction

Let $\Omega \subset \mathbb{R}^n$ be a domain (i.e. open and connected) with smooth boundary $\partial \Omega$. Let $1 \leq n \leq 6$ and $f \in C^{0,\alpha}(\overline{\Omega}) \cap L^2(\Omega)$ be a nonzero given function. We consider the BVP

$$-\Delta u = u^2 + f \quad \text{in } \Omega, \quad u = 0 \quad \text{on } \partial \Omega.$$  \hspace{1cm} (1.1)

The variational or the weak formulation of (1.1) and (1.2) is to find $u \in H^1_0(\Omega)$ such that

$$\int_{\Omega} \nabla u . \nabla v = \int_{\Omega} u^2 v + \int_{\Omega} f v, \quad \text{for all } v \in H^1_0(\Omega).$$  \hspace{1cm} (1.3)

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