Special Solutions to Rotating Stratified Boussinesq Equations

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Abstract: In this paper, we have obtained some special solutions of rotating stratified Boussinesq equations and reduced these equations into the system of six coupled nonlinear ODEs. Further, in absence of strain field we have proved that the reduced system of six coupled ODEs is completely integrable.

Keywords: rotating stratified Boussinesq equation, completely integrable systems, special solutions

Mathematics Subject Classification (2000): 34A05, 35J35.

1 Introduction

The stratified Boussinesq equations form a system of PDEs modelling the movements of planetary atmospheres. It may be noted that the Boussinesq approximation in the literature is also referred to as the Oberbeck–Boussinesq approximation for which, the reader is referred to an interesting article of Rajagopal et al [1] providing a rigorous mathematical justification as perturbations of the Navier–Stokes equations. Majda & Shefter [2] have chosen certain special solutions of this system of PDEs to demonstrate onset of instability when the Richardson number is less than 1/4. In their study of instability in stratified fluids at large Richardson number, Majda & Shefter [2] have obtained the exact solutions to stratified Boussinesq equations neglecting the effects of rotations and viscosity. In his monograph Majda [3] has obtained the special solution of rotating stratified Boussinesq equations excluding the effects of viscosity and finite rotation. Whereas, in this paper we include the effect of rotation. And then we systematically deploy the procedure of Majda & Shefter [2] (as well as the procedure applied by Craik & Criminale in their paper [4]) to obtain the exact solutions of rotating stratified Boussinesq equations and derive the system of six coupled ODEs. Further, in the absence of strain field we proved that the reduced system of ODEs is completely integrable and admits the similar results obtained by Srinivasan et al [5]. For the similar kind of work reader may refer Maas [8, 9].

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