



PERSONAGE IN SCIENCE

Professor V.I. Zubov

to the 80th Birthday Anniversary

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The paper presents a biographical sketch and a review of scientific achievements of Vladimir Ivanovich Zubov (1930–2000), the outstanding researcher in Control and Stability Theory of the 20th century.

1 Brief Outline of Zubov's Life

Vladimir Ivanovich Zubov was born on April 14, 1930 in the town of Kashira, Moscow region, Russia. In 1945 he finished a secondary school.

At the age of 14, Vladimir suffered in the explosion accident happened when he and his playfellows found a hand grenade. His eyes were injured and he failed eyesight soon. In 1949 he finished the Leningrad special school for blind and visually impaired children being the winner of the 15th Leningrad mathematical Olympiad for graduates. The same year he entered the Mathematical and Mechanical Faculty of the Leningrad State University. In 1953 he graduated from the university with honors and received his MSc degree in Mathematics. In the same year he began his post-graduate studies.

Zubov was an active participant of the seminar held under the supervision of Professor N. P. Erugin at the Department of Differential and Integral Equations of the Leningrad State University. When discussing the state of the theory of motion stability N. P. Erugin formulated a set of problems requiring constructive solutions. In particular, very important problems were those of the Lyapunov theorems inversion and representation of the general solution for the differential equations system in the asymptotic stability region. V. I. Zubov obtained a number of profound results in these directions which laid the foundation to his PhD thesis titled “Boundaries of the Asymptotic Stability Region” defended in 1955 (with N. P. Erugin as an advisor and professors E.A. Barbashin and N. N. Krasovskij as official opponents).

In December 1955 Zubov joined the Institute of Mathematics and Mechanics of the Leningrad State University as a leading researcher.

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V. I. Zubov defended his Doctor of Science thesis in April 1960 at the Leningrad Polytechnic Institute. The thesis was based on his book “The methods of A. M. Lyapunov and their application” published in 1957. This book manifested new ideas and fundamental results on the Lyapunov methods and gave rise to constructive approaches for solving various practical problems.

Zubov was affiliated with the Scientific Research Institute of Mathematics and Mechanics of the Leningrad State University as a chief researcher since 1955 till 1962. In 1962 he became the chief of the Laboratory of Control Devices and since 1967 till the end of his life he was the head of the Control Theory Department of the Faculty of Applied Mathematics of the Leningrad (St. Petersburg) State University.

V.I. Zubov married Alexandra Zubova in 1953. His wife is DSc and professor. The Zubovs have 6 children and 21 grandchildren.

2 Basic Trends of His Scientific Work

2.1 Region of asymptotic stability

One of the well known Zubov’s results is his theorem on the region of asymptotic stability. This theorem not only solves the stated problem but is also of immense practical value for engineers and specialists in control theory. The starting point for Zubov’s investigations was the monograph of A. M. Lyapunov “General problem on the stability of motion”. In the 50es of the last century Zubov and other scientists proved the existence of the Lyapunov functions in the cases of stability, asymptotic stability and instability of unperturbed motions. These results ground the possibility of finding the Lyapunov functions for solving the stability problem for various classes of differential equation systems.

V. I. Zubov was the first to solve the problem on estimation of the set of initial values belonging to the attraction region of the asymptotically stable zero solution of ordinary differential equations system. He deduced the equation for the Lyapunov function which allowed the boundary of an asymptotic stability region to be found. In the analytical case the solution for this equation can be obtained in the form of series. On this base the numerical methods were developed for the estimation of an asymptotic stability region. For controllable dynamical systems it was shown that the region of asymptotic stability would be maximized when the optimal stabilizing control was used.

The development of these results was described in his monographs [1, 2, 3, 5, 31].

2.2 Stability of nonlinear systems in critical cases

V. I. Zubov proved that if the zero solution of a system of differential equations with the homogeneous right-hand sides was asymptotically stable, then for this system there existed an homogeneous Lyapunov function satisfying the conditions of the Lyapunov asymptotic stability theorem. He showed that this function could be found as a solution of a special system of partial differential equations [1, 5].

Using the results obtained he estimated the time of transients for asymptotically stable homogeneous systems. Besides, he determined the stability and the ultimate boundedness criteria for nonlinear systems based on the the first homogeneous approximation. Furthermore, new stability conditions were established in the critical cases of several zero roots and of several pairs of pure imaginary roots of characteristic equation. Moreover, he extended the results above to the systems with generally homogeneous right-hand

sides and to the problem of stability by generally homogeneous first approximation [1, 2, 3, 5].

V. I. Zubov also stated a problem of the stability by the first, in the broad sense, approximation, and obtained a number of results for solution of this problem. He investigated the conditions for stability of the zero solution for the arbitrary admissible functions included in the first approximation [27, M].

V. I. Zubov suggested an approach for the construction of solutions of systems of nonlinear equations in a neighborhood of a regular critical point. This problem was stated in the works by Briot and Bouquet with its special cases investigated by Poincaré and Picard. V. I. Zubov solved completely this problem in a classical statement [1, 5].

2.3 Control theory

Zubov's results on the theory of optimal control systems and on solution of the corresponding theoretical and numerical problems of optimal stabilization are presented in his books [2, 3, 6, 13, 17, 18, 20]. He suggested constructive analytical methods for finding the optimal controls. V. I. Zubov established the relationship between the Lyapunov direct method and the theory of optimal control. He introduced the notion of optimal control with respect to the damping of a deviation measure of a transient from the pre-assigned motion. He solved the problem of a synthesis of optimal control. In particular, the problem on synthesis of linear controls with the aftereffect and in the presence of intermediate control points was solved.

Zubov's theorem on the canonical decomposition of nonlinear force fields into the potential component and the gyroscopic one should be mentioned especially [D]. He applied this result to the control problems for finite-dimensional holonomic mechanical systems.

2.4 Asymptotic equilibrium states and asymptotic auto-oscillations

V. V. Nemytskij stated the problem of studying the solutions of differential equations systems for which limit manifolds exist under unbounded increase and decrease of time, but these limit sets are not invariant sets of the systems under consideration. V. I. Zubov showed that in a number of cases such a behavior of motions resulted in the appearance of asymptotic equilibrium states and investigated the conditions for this. It was proved that asymptotic equilibrium states could occur in the systems of differential equations subjected to perturbations tending to zero under the increase of time. Besides that, he established that the forced oscillations arising in perturbed system could be damped if the perturbation was characterized as an oscillatory process with frequency growing in time. In this case the amplitudes of these perturbations can remain finite and, moreover, they can be arbitrary large.

V. I. Zubov also investigated the problem on conservation of auto-oscillations under the action of perturbations formulated by A. A. Andronov. He determined the conditions under which trajectories of perturbed systems tended asymptotically to auto-oscillating modes of the initial systems. He referred to these limiting operating modes as the asymptotic auto-oscillations.

The results obtained along this topic have been presented in monographs [24, 27].

2.5 Nonlinear oscillations and stability of orbits

One of the main directions of Zubov's investigations was the analysis of stationary oscillations of nonlinear systems. He studied the problems of the existence of stationary modes, developed the methods for the construction of these modes and for the analysis of the integral curves behavior in their neighborhoods.

In Zubov's works the approaches for proper and forced oscillations construction for multiple degrees-of-freedom systems were developed. In addition to the well known small parameter method he also introduced the method of successive approximations applied for these investigations. In a number of cases the latter enabled one to obtain more complete results on the appearance of periodic and almost periodic motions and their interconnection.

V. I. Zubov established a qualitative criterion of periodic and almost periodic convergence for nonlinear systems. The constructive approach for the verification of conditions of this criterion was based on the usage of special functions similar to those introduced by A. M. Lyapunov for the stability analysis.

V. I. Zubov also developed a new method for the investigation of integral curves behavior in the neighborhood of a periodical orbit. This method is based on the transformation of the original system into a special form describing the behavior of a mapping point on the hyperplane normal to the periodical orbit. In the framework of this approach new results on the Lyapunov stability of periodical solutions were obtained. Furthermore, the necessary and sufficient conditions were found for the prescribed periodical solution to be the auto-oscillating one. Application of the Zubov method to the differential equation systems possessing several periodical orbits allowed one to simplify the solution of analytical problems in various applications. For instance, new equations of the celestial mechanics were deduced.

Numerous results obtained along this line were presented in monographs [2, 3, 4, 16, 24, 28].

2.6 Development of the dynamical systems theory and analytical representation of stationary oscillations

V. I. Zubov was the first to introduce the concept of a general dynamical system in metric space. He extended the problem of stability investigation for individual trajectories to the problem of stability analysis of invariant sets of dynamical systems. In his works the qualitative structure of a neighborhood of a stable invariant set was studied. Also, he extended the direct Lyapunov method to solution of the problems of stability of invariant sets for general dynamical systems. He also developed a method for estimating the distance from the motion to the invariant set and proved the theorem on the asymptotic stability region for uniformly asymptotically stable and uniformly attracting invariant sets. Furthermore, the method for the determination of boundary of asymptotic stability region was suggested. A special attention was paid to the construction of the theory of periodical dynamical systems. The results obtained were applied for the stability analysis of partial differential equations systems.

One of the most important problems of the theory of dynamical systems is the analysis of stationary oscillations. G. D. Birkhoff proved that the most general class of stationary oscillations of dynamical systems could be described by recurrent functions. V. I. Zubov developed the analytical theory for the representation of the ergodic classes of recurrent functions. He showed that the space of recurrent functions was complete, but neither

linear nor transitive. The approach suggested by Zubov was based on the decomposition of the recurrent functions space into the isolated classes of functions possessing relatively dense sets of common quasiperiods. To implement such a decomposition he generalized the Kronecker theorem on the existence of common solutions for inequalities systems. For the constructed ergodic classes of recurrent functions V. I. Zubov proved the theorem on the approximation of functions from the given class by the trigonometric polynomials of a special form. This theorem generalizes the well known Weierstrass theorem. Moreover, he developed the mathematical methods for the analytical representation of the stable Poisson motions.

The results obtained in this area appeared in [1, 4, 5, 16, 19, 20, 23, 24, 28, B, C, J].

2.7 Systems with aftereffect. Quantization of orbits

Another direction of Zubov's investigations deals with the estimation of the finite velocity of interactions extension or the allowance for the control signal delay in the feedback channel [13, 18, A]. This necessitates the stability analysis of delay-differential systems. V. I. Zubov established the representation of solutions of linear delay-differential systems whose right-hand sides were given by the Stieltjes integrals in the form of asymptotic series. He obtained the root criteria of exponential stability for delay systems.

V. I. Zubov formulated a universal law for the orbits quantization [22, 25, K]. This law is based on taking into account the finiteness of interactions extension velocity by the introduction of delays in the force fields determining the motions of the mass points system. He showed that finiteness of the velocity of the interactions and perturbations extension caused the quantization of orbits of individual mass points and of their configurations. Moreover, the quantization also occurs for the energy levels and for the momenta of momentum.

2.8 Conservative methods of numerical integration

V. I. Zubov developed conservative methods for numerical integration of differential equations systems [15, 17, L, N]. These methods are based on the construction of finite-difference schemes preserving certain properties of motions such as integrals of motion, integral invariants and other physical and qualitative characteristics. Zubov's approach consists in a modification of the known numerical methods by introducing the control in the computation process. This control is constructed with the aim to provide convergence, required precision and stability for the modified numerical method and, in addition, to preserve the given properties of motion on the discrete trajectories. Although the finite-difference equations obtained are the nonlinear and implicit ones, the advantage of such schemes over the known schemes by Euler, Runge and Kutta, and Adams et al. consists in the opportunity they provide for the qualitative behavior analysis of trajectories of the generating differential equations.

2.9 Investigation of rotation motion of a rigid body

V. I. Zubov established that in the Euler and Lagrange cases all motions of a rigid body are periodical or almost periodical with the exception of the motions occurring in a special integral manifold. He determined the precise bounds of nutational oscillations of the proper rotation axis for the dynamically asymmetric rigid body moving inertially

around a fixed point. Furthermore, he established stability and instability conditions of the rigid body motions with respect to axes orientation in the space.

V. I. Zubov developed the methods for the rotational motion control solving the problem of the system transfer from one state to the other. In particular, the problem of the body orientation in a prescribed direction and the problem of scanning the body axes in accordance with the pre-specified program were solved. These methods are based on finding the motions of the carried bodies which create Coriolis forces moments providing the prescribed motion of the carrying body.

For the bodies with the liquid-filled cavities and bodies with the flexible constructions the mathematical models based on the ordinary differential equations were suggested. For such models the analytical constructions of controls providing given rotational motions of the carrier were also obtained.

The development of these methods was described in [7, 13, 15, 20, 22, E].

2.10 Investigation of free and forced oscillations of gyroscopic systems

V. I. Zubov developed a precise method for the analysis of equations of gyroscopic systems motions based on the construction of convergent functional series expansions in powers of the angular momentum inverse. By the use of such series the solutions of linear and nonlinear differential equations systems describing free and forced oscillations of gyroscopic systems were obtained. Furthermore, the stability and asymptotic stability conditions for equilibrium states of gyroscopic systems were deduced and the approach for the numerical solution of stability problem was suggested.

The results obtained in terms of the precise Zubov method were compared with those obtained with the aid of the approximate precession theory. The cases were detected where the latter yielded qualitatively false conclusions on the behavior of oscillations in gyroscopic systems. For the cases where the precession theory results were correct, the method was suggested for the successive refinement of the quantitative results obtained in the framework of this theory.

The results obtained along this topic have been presented in [8, 22].

2.11 Theory of charged particles beams and relativity principles

V. I. Zubov solved the inverse problem of electrodynamics: for the given velocity field of charged particles he proposed a method for the determination of the electric and magnetic fields strengths providing this field. He found the equations for the various fields of such a type and established the theorem on universality of electrodynamics equations. The results obtained were used for the design of various types of electro-physical equipment.

V. I. Zubov treated an arbitrary vector wave as a superposition of a finite number of simple waves. From this point of view the only characteristic of a simple wave turns out to be its phase depending on the time and space coordinates and satisfying the wave equation. He extended Einstein's notion of the equivalence of two coordinate systems based on the relativity principle. This extension allowed a set of relativity principles to be obtained.

Numerous results obtained along this line have been presented in [16, 20, 24, 27, 28, G].

2.12 Distribution of resources and funds

V. I. Zubov developed the theory of a support plan resulted in a subsequent software implementation [9, 21]. The mathematical model construction permits to connect initial, intermediate and final states of the developing branches of a national economy and to solve problems of initial distribution of investments in urban branches of a national economy with an opportunity for redistribution in emergency cases.

As a member of the inter-branch council of scientists V. I. Zubov created the theory of the balanced co-development of various branches of live-stock farming with allowance for the soil and climatic conditions and density of population in a region [20, F].

2.13 Investigation of distribution functions spaces

V. I. Zubov established that any continuous distribution function could be approximated in the real axis with an arbitrarily given precision in the uniform metric by the mixture of normal distributions with the distinct expected values and variances. Furthermore, he showed that the normal distribution law is not of a unique nature. It was proved that any continuous distribution law gave a set of sliding sums with weight coefficients defining an everywhere dense subset in the space of all continuous distribution functions [H, I, O].

3 Applied Investigations

Zubov's investigations were always aimed at applications. Since 1957 he was efficiently contributing to the development of modern technologies in the following fields:

- (1) inertial navigation systems for which he solved the problem on deviation of the gyro-system axes depending on nutational oscillations and kinematic moment of inertia of gyro-rotors;
- (2) self-guidance of cruise missiles;
- (3) precision control systems of spacecraft position for the "Proton" system;
- (4) control systems for the rotational motion of spacecrafts for the precision orientation of sensitive axes of devices on the base of magneto-hydrodynamic control systems with the use of conducting fluids in feedback contours;
- (5) control problems for beams of charged particles to be transported in a given physical channel;
- (6) noise stability of the information transmission methods;
- (7) tactical scheme constructions for the USSR Navy to oppose aircraft carriers of a potential enemy.

In all the above mentioned pure applied directions Zubov obtained fundamental results in control and stability theory.

On the occasion of awarding V. I. Zubov by the USSR State Prize, the president of the Academy of Sciences of the Soviet Union M. V. Keldysh noted: "Zubov's works are well known in the Soviet Union and abroad. The profound researches carried out by him on the theory of motion stability, theory of automatic control and theory of optimal processes allow one to solve the important applied problems, in particular, in the field of design of controlled automatic devices and stabilization of program motions. Zubov's methods are also effective in the application to control problems arising in industry, mathematical economics, biology, medicine and navigation".

4 Science Management and Teaching Activity

Fundamental successes in the investigation of new branches of applied mathematics and control theory resulted in the creation of the Laboratory of Control Devices in 1962, the Control Theory Department in 1967, the Faculty of Applied Mathematics and Control Theory in 1969 and the Research Institute of Computational Mathematics and Control Processes at the Leningrad (St. Petersburg) State University in 1971. Furthermore, V. I. Zubov organized the Center of Applied Mathematics and Control Processes. In his time V. I. Zubov headed these institutions. Also, he was a permanent Chief of the faculty Curriculum Committee and the Special Council for DSc Dissertation defenses. He was an advisor for 20 DSc and about 100 PhD dissertations. Under Zubov's supervision a worldwide known school in control theory was developed in St. Petersburg.

5 Editorial Activity and International Scientific Activity

For many years V. I. Zubov was a member of the Editorial Board of the Journal of "Differential equations".

He was chair of the Program Committees of the International Seminars "Beam Dynamics & Optimization", the International Symposium "Hydrogen Energetic, Theoretical and Engineering Solutions", the 11th International IFAC Seminar "Control Applications of Optimization".

6 Awards

In 1968 V. I. Zubov became the USSR State Prize winner for his pioneer works in Control Theory. Twice, in 1962 and 1996 he received the Leningrad (St. Petersburg) University Prize for scientific achievements. In 1981 he was elected as corresponding member of the Soviet Union Academy of Sciences and in 1998 he was conferred with a title of Honored Scientist of the Russian Federation.

In 1996 Zubov's scientific school of "Processes of control and stability" was the winner of the competition for the state support of the leading scientific schools of Russia.

In 2001, the Research Institute of Computational Mathematics and Control Processes of St. Petersburg State University was named after him.

For his outstanding scientific merits Zubov's name was perpetuated as the name of minor planet 'ZUBOV 10022'. This asteroid has the size of 6 km, the brightness of 13.8 magnitude, and the largest orbit semi-axis of 2.369 astronomical units.

7 Public Activities

In addition to his intensive scientific researches and tuition duties, V. I. Zubov was involved in public social activity. He was the President of the St. Petersburg Charity Foundation for blind and visually impaired children.

His poetic talent is evident in his books "Behest of the past generations", St. Petersburg: "Mobil'nost Plus" Publisher, 1993 and "Poetry. Sonnets. Behest of the past generations", St. Petersburg: St. Petersburg State University Publisher, 2000.

V. I. Zubov is the author of about 200 publications including 31 monographs and books. Four of his monographs were republished abroad in English and French.

8 List of Monographs and Books by V. I. Zubov

- [1] Methods of A. M. Lyapunov and their Application. Leningrad, Leningrad State University, 1957, 242 p. (Russian)
- [2] Mathematical Methods of the Study of Automatic Control Systems. Leningrad, Sudpromgiz, 1959, 324 p. (Russian)
- [3] Mathematical Methods of the Study of Automatic Control Systems. New York etc.: Pergamon Press; Yerusalem: Academic Press, 1962, 327 p.
- [4] Oscillations in Nonlinear and Controlled Systems. Leningrad, Sudpromgiz, 1962, 630 p. (Russian)
- [5] Methods of A. M. Lyapunov and Their Applications, Groningen: NoordHoff Ltd., 1964, 263 p.
- [6] Theory of Optimal Control of Ships and Other Moving Objects. Leningrad, Sudpromgiz, 1966, 352 p. (Russian)
- [7] Dynamics of Free Rigid Body and Determination of Its Orientation in the Space. Leningrad, Leningrad State University, 1968, 208 p. (Russian, with V. S. Ermolin et al.)
- [8] Analytical Dynamics of Gyroscopic Systems. Leningrad, Sudostroenie, 1970, 317 p. (Russian)
- [9] The Problem of Optimal Distribution of Capital Investments. Leningrad, Leningrad State University, 1971, 26 p. (Russian, with L. A. Petrosyan)
- [10] Lectures on Control Theory, Part 1. Leningrad, Leningrad State University, 1972, 203 p. (Russian)
- [11] Stability of Motion. Methods of Lyapunov and their Application. Moscow, Vysshaya Shkola, 1973, 271 p. (Russian)
- [12] Mathematical Methods of the Study of Automatic Control Systems, 2nd ed. Leningrad, Mashinostroenie, 1974, 335 p. (Russian)
- [13] Lectures on Control Theory. Moscow, Nauka, 1975, 496 p. (Russian)
- [14] Theorie de la Commande. Moscow, Mir, 1978, 470 p. (French)
- [15] Control of Rotational Motion of a Rigid Body. Leningrad, Leningrad State University, 1978, 200 p. (Russian, with V. S. Ermolin et al.)
- [16] Oscillations Theory. Moscow, Vysshaya Schkola, 1979, 400 p. (Russian)
- [17] Stability Problem of Control Processes. Leningrad, Sudostroenie, 1980, 253 p. (Russian)
- [18] Theory of Equations of Controlled Motion. Leningrad, Leningrad State University, 1980, 288 p. (Russian)
- [19] Stability of Invariant Sets of Dynamical Systems. Saransk, Mordovian State University, 1980, 80 p. (Russian)
- [20] Dynamics of Controlled Systems. Moscow, Vysshaya Schkola, 1982, 285 p. (Russian)
- [21] Mathematical Methods in Planning. Leningrad, Leningrad State University, 1982, 80 p. (Russian, with L. A. Petrosyan)
- [22] Analytical Dynamics of Systems of Bodies. Leningrad, Leningrad State University, 1983, 343 p. (Russian)
- [23] Periodical Dynamical Systems. Saransk, Mordovian State University, 1983, 88 p. (Russian)
- [24] Oscillations and Waves. Leningrad, Leningrad State University, 1989, 416 p. (Russian)

- [25] Mathematical Problem of Quantization. Saransk, Saransk Division of Saratov University, 1989, 56 p. (Russian)
- [26] Mathematical Theory of Motion Stability. St. Petersburg, AO "Mobil'nost Plus", 1997, 340 p.
- [27] Processes of Control and Stability. St. Petersburg, St. Petersburg State University, 1999, 325 p. (Russian)
- [28] Theory of Oscillations. Singapore etc., World Scientific, 1999, 400 p.
- [29] Stability Problem of Control Processes, 2nd ed. St. Petersburg, St. Petersburg State University, 2001, 354 p. (Russian)
- [30] Dynamics of Controlled Systems, 2nd ed. St. Petersburg, St. Petersburg State University, 2004, 380 p. (Russian)
- [31] Boundaries of the Asymptotic Stability Region. St. Petersburg, AOOT "Mobil'nost Plus", 2007, 85 p. (Russian)

9 List of the Zubov Selected Papers

- [A] On the theory of linear stationary systems with delay. *Izvestija Vuzov. Matematika* (6) (1958) 86–95 (Russian).
- [B] On the ergodic classes of recurrent motions. *Doklady Akademii Nauk SSSR* **132**(3) (1960) 507–509. (Russian)
- [C] On the recurrent functions theory. *Sibirskij Matematicheskij Zhurnal* **3**(4) (1962) 532–560. (Russian)
- [D] Canonical structure of a vector force field. In: *Problems of Mechanics of a Solid Deformable Body*. Leningrad, Sudostroenie, 1970, 167–170. (Russian)
- [E] On the active control of rigid body rotation. *Differentsial'nye Uravneniya* **6**(11) (1970) 2086–2087. (Russian)
- [F] Modeling of biological processes by the use of differential equations, In: *Problems of Cybernetics, Issue 25, Part 2: Biotechnical Systems*, Moscow, 1975, 3–9. (Russian)
- [G] On the control of charge particles in magnetic field. *Doklady Akademii Nauk SSSR* **232**(4) (1977) 798–799. (Russian)
- [H] A finite-dimensional characterization of probability distributions. *Doklady Akademii Nauk SSSR* **314**(2) (1990) 283–287. (Russian)
- [I] Interpolation and approximation of probability distributions. *Doklady Akademii Nauk SSSR* **316**(6) (1991) 1298–1301. (Russian)
- [J] Analytical representation of motions that are Poisson stable. *Doklady Akademii Nauk SSSR* **322**(1) (1992) 28–32. (Russian)
- [K] Independence of evolutionary development of species from aftereffects. *Doklady Akademii Nauk SSSR* **323**(4) (1992) 632–635. (Russian)
- [L] Conservative methods of numerical integration of motions equations for the charge particles beams. *Doklady Akademii Nauk Rossii* **345**(2) (1995) 161–163. (Russian)
- [M] Asymptotic stability by the first, in the broad sence, approximation. *Doklady Akademii Nauk Rossii* **346**(3) (1996) 295–296. (Russian)
- [N] Conservative numerical methods for differential equations integration in nonlinear mechanics. *Doklady Akademii Nauk Rossii* **354**(4) (1997) 446–448. (Russian)
- [O] Random variables and stochastic processes. In: *Proceedings of the 11th IFAC Workshop "Control Applications of Optimization (CAO'2000)", St. Petersburg, Russia, July 3–6, 2000*, Great Britain: Pergamon Press, Vol. 1, 403–414.